Questions: The scalar product

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Summary

A selection of questions for the study guide on the scalar product

*Before attempting these questions, it is highly recommended that you read* [*Guide: The scalar product*](../studyguides/scalarproduct.qmd)*, as well as* [*Guide: Introduction to quadratic equations*](../studyguides/introtoquadratics.qmd)*.*

## Q1

Find the scalar product of $a$ and $b$.

1.1. $a=\left(\begin{matrix}6\\3\\4\end{matrix}\right)$ and $b=\left(\begin{matrix}1\\4\\2\end{matrix}\right)$

1.2. $a=\left(\begin{matrix}10\\−7\\4\end{matrix}\right)$ and $b=\left(\begin{matrix}3\\−5\\13\end{matrix}\right)$

1.3. $a=\left(\begin{matrix}−44\\−12\\3\end{matrix}\right)$ and $b=\left(\begin{matrix}61\\−25\\93\end{matrix}\right)$

1.4. $a=\left(\begin{matrix}54\\38\\0\end{matrix}\right)$ and $b=\left(\begin{matrix}32\\−55\\13\end{matrix}\right)$

1.5. $a=2i+7j+k$ and $b=6i+4j+8k$

1.6. $a=−3i+10j−8k$ and $b=i−12j+9k$

1.7. $a=17j+23k$ and $b=6i−23j−8k$

1.8. $a=i$ and $b=j$.

What can you say about the result of 1.8.? Can you deduce similar conclusions for the scalar product of different combinations of the vectors $i$, $j$, $k$?

## Q2

Using the geometric definition of the scalar products, find the smallest angle $θ$ in between $a$ and $b$ in degrees. If your answer is not a whole number, give your answer to an accuracy of one decimal place.

2.1. $a=\left(\begin{matrix}−5\\2\\−3\end{matrix}\right)$ and $b=\left(\begin{matrix}2\\−2\\11\end{matrix}\right)$

2.2. $a=\left(\begin{matrix}1\\1\\1\end{matrix}\right)$ and $b=\left(\begin{matrix}1\\−1\\1\end{matrix}\right)$

2.3. $a=\left(\begin{matrix}−8\\1\\−4\end{matrix}\right)$ and $b=\left(\begin{matrix}−1\\−5\\7\end{matrix}\right)$

2.4. $a=\left(\begin{matrix}1.2\\−1.4\\−3.1\end{matrix}\right)$ and $b=\left(\begin{matrix}−5.4\\9.7\\−7.5\end{matrix}\right)$

2.5. $a=\left(\begin{matrix}45\\65\\54\end{matrix}\right)$ and $b=\left(\begin{matrix}−19\\−58\\71\end{matrix}\right)$

2.6. $a=\left(\begin{matrix}1\\0\\0\end{matrix}\right)$ and $b=\left(\begin{matrix}0\\0\\1\end{matrix}\right)$

2.7. $a=\left(\begin{matrix}−1\\−2\\3\end{matrix}\right)$ and $b=\left(\begin{matrix}4\\−5\\6\end{matrix}\right)$

2.8. $a=\left(\begin{matrix}−17\\3\\8\end{matrix}\right)$ and $b=\left(\begin{matrix}12\\−19\\−16\end{matrix}\right)$

## Q3

Find the value(s) of $λ$ for which $a$ and $b$ are perpendicular.

3.1. $a=\left(\begin{matrix}2\\4\\7\end{matrix}\right)$ and $b=\left(\begin{matrix}1\\λ\\−2\end{matrix}\right)$

3.2. $a=\left(\begin{matrix}0\\1\\λ\end{matrix}\right)$ and $b=\left(\begin{matrix}1\\2\\3\end{matrix}\right)$

3.3. $a=\left(\begin{matrix}9\\−2\\11\end{matrix}\right)$ and $b=\left(\begin{matrix}λ\\−λ\\3\end{matrix}\right)$

3.4. $a=\left(\begin{matrix}λ\\6\\1\end{matrix}\right)$ and $b=\left(\begin{matrix}λ\\λ\\8\end{matrix}\right)$

3.5. $a=\left(\begin{matrix}−2λ^{2}\\4\\14\end{matrix}\right)$ and $b=\left(\begin{matrix}3\\2λ\\1\end{matrix}\right)$

3.6. $a=\left(\begin{matrix}−5\\9\\2λ\end{matrix}\right)$ and $b=\left(\begin{matrix}λ\\−2\\λ\end{matrix}\right)$

3.7. $a=\left(\begin{matrix}−7\\4\\2λ\end{matrix}\right)$ and $b=\left(\begin{matrix}2λ\\1\\6λ\end{matrix}\right)$

3.8. $a=\left(\begin{matrix}−25\\−λ^{2}\\−2\end{matrix}\right)$ and $b=\left(\begin{matrix}3λ\\−11\\7\end{matrix}\right)$

[After attempting the questions above, please click this link to find the answers.](../answers/as-scalarproduct.qmd)

## Version history and licensing

v1.0: initial version created 08/23 by Ritwik Anand as part of a University of St Andrews STEP project.

* v1.1: edited 05/24 by tdhc.

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