Questions: PMFs, PDFs, and CDFs

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Summary

A selection of questions to test your understanding of probability mass functions (PMFs), probability density functions (PDFs), and cumulative distribution functions (CDFs).

Before attempting these questions it is highly recommended that you read Guide: PMFs, PDFs, and CDFs.

Q1

For each of the scenarios below, determine if the given distribution is a valid PMF and answer the following questions.

1.1.

Let X be the random variable representing the result of rolling a biased four sided-die. The PMF of X is given by:

x	1	2	3	4
$P(X = x)$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{2}$	$\frac{1}{5}$

What is $P(X = 4)$?

1.2.

A discrete random variable X has five possible outcomes $(1, 2, 3, 4,$ or $5)$, and the PMF is given by:

x	1	2	3	4	5
$P(X = x)$	0.25	0.35	0.05	0.2	0.1

What is the probability of $X = 3$ or $X = 4$?

1.3.

A coin is tossed, where the probability of tails is 70 and heads is 30. Let X represent the result of the coin toss. Complete the table below:

x	Heads	Tails
$P(X = x)$		

1.4.

A discrete random variable X has the possible outcomes $1, 2, 3, 4, 5, 6$, or 7, with the following PMF:

x	1	2	3	4	5	6	7
$P(X = x)$	0.1	0.05	0.05	0.3	0.25	0.75	0.35

Is this a valid PMF? Justify your answer either way.

1.5.

A bag contains 5 red, 3 blue, and 2 green sweets from a sweet shop. Let X represent the color of a randomly picked sweet:

(a) What is the probability of picking a blue sweet?

(b) Construct the PMF for this scenario by completing the table:

x	Red	Blue	Green
$P(X = x)$			

1.6.

The PMF for a random variable X is given as:

x	1	2	3	4
$P(X = x)$	p	$2p$	$3p$	$4p$

(a) For what value of p is this a valid PMF?

(b) For this value of p, what is $P(X = 3)$?

Q2

For each of the scenarios below, determine if the given distribution is a valid PDF and answer the following questions.

2.1.

Let X be a continuous random variable on the interval $[0, 2]$ with the PDF:

$$f(x) = \begin{cases} \frac{1}{2} & \text{if } 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

What is the probability that X lies between 1 and 2?

2.2.

Let X be a continuous random variable with the PDF:

$$f(x) = \begin{cases} 2x & \text{if } 0 \leq x \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

(a) What is the probability that X lies between 0.5 and 1?

(b) What is $P(0.25 \leq X \leq 0.75)$?

2.3.

Let X be a continuous random variable uniformly distributed between 3 and 7. The PDF is:

$$f(x) = \begin{cases} \frac{1}{4} & \text{if } 3 \leq x \leq 7 \\ 0 & \text{otherwise} \end{cases}$$

What is the probability that X lies between 3 and 6?

2.4.

The PDF of a random variable X is given by:

$$f(x) = \begin{cases} \frac{1}{9} & \text{if } 1 \leq x \leq 4 \\ \frac{1}{4} & \text{if } 5 \leq x \leq 7 \\ 0 & \text{otherwise} \end{cases}$$

Is this a valid PDF? Justify your answer either way.

2.5.

Consider the PDF:

$$f(x) = \begin{cases} kx^2 & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) For what value of k is this a valid PDF?

(b) For this value of k, what is $P(0.2 \leq x \leq 0.3)$?

2.6.

The PDF of X is given by:

$$f(x) = \begin{cases} 4x & \text{if } 0 \leq x < 0.5, \\ 4 - 4x & \text{if } 0.5 \leq x < 0.75, \\ 0.5 & \text{if } 0.75 \leq x \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

Is this a valid PDF? Justify your answer either way.

Q3

For each of the scenarios below, answer the following questions.

3.1.

In a scenario involving a discrete random variable, the following CDF is given:

x	1	2	3	4
$P(X = x)$	0.1	0.3	0.5	1

(a) What is $F(3)$?

(b) What is $P(X > 2)$?

3.2.

For the random variable uniformly distributed on $[0, 2]$ as seen in Q2.2:

(a) Calculate the CDF at values 0.5, 1, and 2.

(b) What is $F(3)$?

3.3.

For the PDF given in Q2.3:

(a) Calculate the CDF at points $x = 4$, $x = 5$, and $x = 6$.

(b) What is $P(X > 5)$?

3.4.

The CDF of X for a scenario is given by:

x	1	2	3	4	5	6
$P(X = x)$	0.1	0.2	0.5	0.4	0.8	1

Is this a valid CDF? Justify your answer either way.

After attempting the questions above, please click this link to find the answers.

Version history and licensing

v1.0: initial version created 12/24 by Sophie Chowgule as part of a University of St Andrews VIP project.