Questions: PMFs, PDFs, and CDFs

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Summary

A selection of questions to test your understanding of probability mass functions (PMFs), probability density functions (PDFs), and cumulative distribution functions (CDFs).

*Before attempting these questions it is highly recommended that you read* [*Guide: PMFs, PDFs, and CDFs*](../studyguides/pmfspdfscdfs.qmd)*.*

## Q1

For each of the scenarios below, determine if the given distribution is a valid PMF and answer the following questions.

#### 1.1.

Let $X$ be the random variable representing the result of rolling a biased four sided-die. The PMF of $X$ is given by:

| $x$ | $1$ | $2$ | $3$ | $4$ |
| --- | --- | --- | --- | --- |
| $P\left(X=x\right)$ | $\frac{1}{10}$ | $\frac{1}{5}$ | $\frac{1}{2}$ | $\frac{1}{5}$ |

What is $P\left(X=4\right)$?

#### 1.2.

A discrete random variable $X$ has five possible outcomes ($1,2,3,4,$ or $5$), and the PMF is given by:

| $x$ | $1$ | $2$ | $3$ | $4$ | $5$ |
| --- | --- | --- | --- | --- | --- |
| $P\left(X=x\right)$ | $0.25$ | $0.35$ | $0.05$ | $0.2$ | $0.1$ |

What is the probability of $X=3$ or $X=4$?

#### 1.3.

A coin is tossed, where the probability of tails is $70$ and heads is $30$. Let $X$ represent the result of the coin toss. Complete the table below:

| $x$ | Heads | Tails |
| --- | --- | --- |
| $P\left(X=x\right)$ |  |  |

#### 1.4.

A discrete random variable $X$ has the possible outcomes $1,2,3,4,5,6,$ or $7$, with the following PMF:

| $x$ | $1$ | $2$ | $3$ | $4$ | $5$ | $6$ | $7$ |
| --- | --- | --- | --- | --- | --- | --- | --- |
| $P\left(X=x\right)$ | $0.1$ | $0.05$ | $0.05$ | $0.3$ | $0.25$ | $0.75$ | $0.35$ |

Is this a valid PMF? Justify your answer either way.

#### 1.5.

A bag contains $5$ red, $3$ blue, and $2$ green sweets from a sweet shop. Let $X$ represent the color of a randomly picked sweet:

1. What is the probability of picking a blue sweet?
2. Construct the PMF for this scenario by completing the table:

| $x$ | Red | Blue | Green |
| --- | --- | --- | --- |
| $P\left(X=x\right)$ |  |  |  |

#### 1.6.

The PMF for a random variable $X$ is given as:

| $x$ | 1 | 2 | 3 | 4 |
| --- | --- | --- | --- | --- |
| $P\left(X=x\right)$ | $p$ | $2p$ | $3p$ | $4p$ |

1. For what value of $p$ is this a valid PMF?
2. For this value of $p$, what is $P\left(X=3\right)$?

## Q2

For each of the scenarios below, determine if the given distribution is a valid PDF and answer the following questions.

#### 2.1.

Let $X$ be a continuous random variable on the interval $\left[0,2\right]$ with the PDF:

$$f\left(x\right)=\left\{\begin{matrix}\frac{1}{2}&if 0\leq x\leq 2\\0&otherwise\end{matrix}\right.$$

What is the probability that $X$ lies between 1 and 2?

#### 2.2.

Let $X$ be a continuous random variable with the PDF:

$$f\left(x\right)=\left\{\begin{matrix}2x&if 0\leq x\leq 1,\\0&otherwise.\end{matrix}\right.$$

1. What is the probability that $X$ lies between $0.5$ and $1$?
2. What is $P\left(0.25\leq X\leq 0.75\right)$?

#### 2.3.

Let $X$ be a continuous random variable uniformly distributed between $3$ and $7$. The PDF is:

$$f\left(x\right)=\left\{\begin{matrix}\frac{1}{4}&if 3\leq x\leq 7\\0&otherwise\end{matrix}\right.$$

What is the probability that $X$ lies between $3$ and $6$?

#### 2.4.

The PDF of a random variable $X$ is given by:

$$f\left(x\right)=\left\{\begin{matrix}\frac{1}{9}&if 1\leq x\leq 4\\\frac{1}{4}&if 5\leq x\leq 7\\0&otherwise\end{matrix}\right.$$

Is this a valid PDF? Justify your answer either way.

#### 2.5.

Consider the PDF:

$$f\left(x\right)=\left\{\begin{matrix}kx^{2}&if 0\leq x\leq 1\\0&otherwise\end{matrix}\right.$$

1. For what value of $k$ is this a valid PDF?
2. For this value of $k$, what is $P\left(0.2\leq x\leq 0.3\right)$?

#### 2.6.

The PDF of $X$ is given by:

$$f\left(x\right)=\left\{\begin{matrix}4x&if 0\leq x<0.5,\\4−4x&if 0.5\leq x<0.75,\\0.5&if 0.75\leq x\leq 1,\\0&otherwise.\end{matrix}\right.$$

Is this a valid PDF? Justify your answer either way.

## Q3

For each of the scenarios below, answer the following questions.

#### 3.1.

In a scenario involving a discrete random variable, the following CDF is given:

| $x$ | $1$ | $2$ | $3$ | $4$ |
| --- | --- | --- | --- | --- |
| $P\left(X=x\right)$ | $0.1$ | $0.3$ | $0.5$ | $1$ |

1. What is $F\left(3\right)$?
2. What is $P\left(X>2\right)$?

#### 3.2.

For the random variable uniformly distributed on $\left[0,2\right]$ as seen in Q2.2:

1. Calculate the CDF at values $0.5$, $1$, and $2$.
2. What is $F\left(3\right)$?

#### 3.3.

For the PDF given in Q2.3:

1. Calculate the CDF at points $x=4$, $x=5$, and $x=6$.
2. What is $P\left(X>5\right)$?

#### 3.4.

The CDF of $X$ for a scenario is given by:

| $x$ | $1$ | $2$ | $3$ | $4$ | $5$ | $6$ |
| --- | --- | --- | --- | --- | --- | --- |
| $P\left(X=x\right)$ | $0.1$ | $0.2$ | $0.5$ | $0.4$ | $0.8$ | $1$ |

Is this a valid CDF? Justify your answer either way.

[After attempting the questions above, please click this link to find the answers.](../answers/as-pmfspdfscdfs.qmd)

## Version history and licensing

v1.0: initial version created 12/24 by Sophie Chowgule as part of a University of St Andrews VIP project.

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