Questions: Multivariate chain rule

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Summary

A selection of questions for the study guide on the multivariate chain rule.

Before attempting these questions, it is highly recommended that you read Guide: Multivariate chain rule.

Q1

Let $z = z(x, y)$ be a function where both x and y depend on an independent variable t.

For each function given below, use the multivariate chain rule or otherwise to find $\frac{\mathrm{d}z}{\mathrm{d}t}$, expressing your answer in terms of t only.

1.1. $z = x^2 y$ where $x = \sin(t)$ and $y = e^{2t}$.

1.2. $z = \ln(xy)$ where $x = t^3$ and $y = \cos(t)$.

1.3. $z = x^3 + y^3$ where $x = \sqrt{t}$ and $y = t^2 + 1$.

1.4. $z = e^{xy}$ where $x = t$ and $y = \ln(t + 1)$.

1.5. $z = x \tan(y)$ where $x = \cos(t)$ and $y = t^2$.

1.6. $z = x^2 + 3xy + y^3$ where $x = 2t - 1$ and $y = 5\sin(t)$.

1.7. $z = \frac{x}{y}$ where $x = t^2 + 1$ and $y = t - 2$.

1.8. $z = \sqrt{x^2 + y^2}$ where $x = \cos(t)$ and $y = \sin(t)$.

1.9. $z = xy^2 + yx^2$ where $x = e^t$ and $y = t^3$.

1.10. $z = \ln(x) + xy$ where $x = t^2$ and $y = e^{-t}$.

1.11. $z = x^2 y$ where $x = 2t$ and $y = \ln(t)$.

1.12. $z = x^2 \sin(y)$ where $x = t^3 + 1$ and $y = 3t$.

1.13. $z = \tan^{-1}\left(\frac{y}{x}\right)$ where $x = t$ and $y = t^2$.

1.14. $z = xe^y$ where $x = \ln(t + 2)$ and $y = \sqrt{t}$.

Q2

Let $z = z(x, y)$ be a function where both x and y depend on two independent variables s and t.

For each function, use the multivariate chain rule to find $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$, expressing your answers in terms of s and t only.

2.1. $z = x^2 y$ where $x = s + t$ and $y = s^2 - t^2$.

2.2. $z = \ln(x + y)$ where $x = e^s \cos(t)$ and $y = e^s \sin(t)$.

2.3. $z = x^3 - 3xy$ where $x = st$ and $y = s + t$.

2.4. $z = e^{x+y}$ where $x = s^2$ and $y = \ln(t)$.

2.5. $z = x \sin(y)$ where $x = s - t^2$ and $y = st$.

2.6. $z = x^2 + y^2$ where $x = \cos(s)\sin(t)$ and $y = \sin(s)\cos(t)$.

2.7. $z = xy + x^2$ where $x = s + t$ and $y = s - t$.

2.8. $z = \ln(x) - \ln(y)$ where $x = s + t$ and $y = st$.

2.9. $z = \tan(x + y)$ where $x = s^2 - t$ and $y = s + t^2$.

2.10. $z = \tan^{-1}\left(\dfrac{y}{x}\right)$ where $x = s^2 - t^2$ and $y = 2st$.

Q3

Let $w = w(x_1, \dots, x_n)$ be a function that depends on variables $x_1, \dots, x_n$, where each x_i is itself a function of $t_1, \dots, t_m$.

For each function, write the appropriate form of the multivariate chain rule and find the resulting partial derivatives.

3.1. $w = x^2 + y^2 + z^2$ where $\begin{cases} x = s + t \\ y = s - t \\ z = st \end{cases}$

3.2. $w = xy + z$ where $\begin{cases} x = s + t + u \\ y = st \\ z = t + u \end{cases}$

3.3. $w = \sin(xy) + \cos(z)$ where $\begin{cases} x = s^2 \\ y = t^2 \\ z = s + t \end{cases}$

3.4. $w = x^2 + y^2$ where $\begin{cases} x = s + t + u \\ y = s - t + u \end{cases}$

After attempting the questions above, please click this link to find the answers.

Version history and licensing

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