Questions: Matrix multiplication with special matrices

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Summary

A selection of questions on matrix multiplication with special matrices.

Before attempting these questions, it is highly recommended that you read Guide: Matrix multiplication with special matrices.

Q1

You are given the following matrix multiplications. Express each of these as a system of simultaneous equations.

1.1. $\begin{bmatrix} 3 & 3 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

1.2. $\begin{bmatrix} 3 & 3 \\ 1 & -1 \\ -4 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 0 \end{bmatrix}$

1.3. $\begin{bmatrix} 3 & 3 & 3 \\ 0 & 1 & -1 \\ -4 & 3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 0 \end{bmatrix}$

1.4. $\begin{bmatrix} 3 & 3 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \end{bmatrix}$

1.5. $\begin{bmatrix} 3 & 3 & 3 & 3 \\ 1 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

Q2

You are given the following systems of simultaneous equations. Express each of these as a matrix multiplication.

2.1.

$$9x - y = -4$$
$$x + y = 1$$

2.2.

$$x + 3y + z = 8$$
$$-x - y + 9z = -1$$

2.3.

$$3x + y = 4$$
$$x - y = 5$$
$$8x - 9y = 1$$

2.4.

$$3x + 3y + 3z = 4$$
$$z = 1$$
$$-4x - 4y + 8z = 1$$

2.5.

$$2x + 2y + 2z + 9t = 4$$
$$-x - y - z - t = -10$$

Q3

Let $S = \begin{bmatrix} 1 & -2 & 5 \\ -3 & 4 & -1 \end{bmatrix}$, $T = \begin{bmatrix} 5 & -6 \\ 7 & 2 \\ 0 & 8 \end{bmatrix}$. Work out the following matrix products.

3.1. $S0_{3\times 9}$

3.2. $0_{9\times 3}T$

3.3. SI_3

3.4. I_2S

3.5. I_2T

Q4

For each of the following statements, give an example of an upper triangular 2×2 matrix A and a lower triangular 2×2 matrix B such that:

4.1. AB is upper triangular.

4.2. AB is lower triangular.

4.3. AB is diagonal.

4.4. AB is the 2×2 identity matrix.

4.5. AB is neither upper nor lower triangular.

After attempting the questions above, please click this link to find the answers.

Version history

v1.0: initial version created 05/26 by tdhc.