Questions: Introduction to partial differentiation

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Summary

A selection of questions for the study guide on the introduction to partial differentiation.

*Before attempting these questions, it is highly recommended that you read* [*Guide: Introduction to partial differentiation*](../studyguides/introtopartialdifferentiation.qmd)*.*

## Q1

Find all possible first-order partial derivatives for each function $f$.

1.1. $ f\left(x,y\right)=x^{2}y+y^{3}$

1.2. $ f\left(x,y\right)=3x^{3}−2y^{4}+xy$

1.3. $ f\left(x,y\right)=ysin\left(2x\right)+3$

1.4. $ f\left(x,y\right)=e^{xy}+2x^{2}y^{3}$

1.5. $ f\left(x,y\right)=ln\left(x\right)+xln\left(y\right)+3x$

1.6. $ f\left(x,y\right)=\frac{y}{x}−\frac{x}{y}$

1.7. $ f\left(x,y\right)=xexp\left(y^{2}\right)$

1.8. $ f\left(x,y\right)=\sqrt{x^{2}+y^{2}}$

1.9. $ f\left(x,y\right)=\left(3x+2y\right)^{4}$

1.10. $ f\left(x,y\right)=ysin\left(xy\right)$

1.11. $ f\left(x,y\right)=sin\left(x^{2}+y^{2}\right)$

1.12. $ f\left(x,y\right)=ln\left(1+x^{2}y^{2}\right)$

1.13. $ f\left(x,y,z\right)=x^{2}ysin\left(z\right)$

1.14. $ f\left(x,y,z\right)=\left(x+y\right)\left(y+z\right)\left(z+x\right)$

1.15. $ f\left(x,y,z\right)=\frac{xyz}{x+y+z}$

## Q2

A function $f\left(x,y\right)$ is called harmonic if it satisfies the equation

$$\frac{∂^{2}f}{∂x^{2}}+\frac{∂^{2}f}{∂y^{2}}=0$$

Show that each of these functions is harmonic by calculating the pure second-order partial derivatives and checking that their sum is zero.

2.1. $ f\left(x,y\right)=x^{2}−y^{2}$

2.2. $ f\left(x,y\right)=xy$

2.3. $ f\left(x,y\right)=x^{3}−3xy^{2}$

2.4. $ f\left(x,y\right)=cos\left(x\right)sinh\left(y\right)$

2.5. $ f\left(x,y\right)=e^{x}sin\left(y\right)$

2.6. $ f\left(x,y\right)=tan^{−1}\left(\frac{y}{x}\right)$

2.7. $ f\left(x,y\right)=ln\left(x^{2}+y^{2}\right)$

## Q3

For each function $f\left(x,y\right)$, calculate the mixed second-order partial derivatives and confirm that they satisfy the equation

$$\frac{∂^{2}f}{∂x∂y}=\frac{∂^{2}f}{∂y∂x}$$

3.1. $ f\left(x,y\right)=x^{2}y+xy^{2}$

3.2. $ f\left(x,y\right)=2x^{2}cos\left(y\right)$

3.3. $ f\left(x,y\right)=\left(x+y\right)^{5}$

3.4. $ f\left(x,y\right)=\frac{x}{1+y}$

3.5. $ f\left(x,y\right)=\sqrt{x^{2}+y^{2}}$

3.6. $ f\left(x,y\right)=x^{2}sin\left(y\right)+y^{2}cos\left(x\right)$

3.7. $ f\left(x,y\right)=tan^{−1}\left(xy\right)$

[After attempting the questions above, please click this link to find the answers.](../answers/as-introtopartialdifferentiation.qmd)

## Version history and licensing

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