Questions: Expected value, variance, standard deviation

Tom Coleman

Summary

A selection of questions to test your understanding of expected values, variance, and standard deviation.

*Before attempting these questions it is highly recommended that you read* [*Guide: Expected value, variance, standard deviation*](../studyguides/expectedvariance.qmd)*.*

## Q1

For each of the following valid random variables with associated probability mass function, work out the expected value and variance.

#### 1.1.

Let $X$ be the random variable representing the result of rolling a biased four sided-die. The PMF of $X$ is given by:

| $x$ | $1$ | $2$ | $3$ | $4$ |
| --- | --- | --- | --- | --- |
| $P\left(X=x\right)$ | $\frac{1}{10}$ | $\frac{1}{5}$ | $\frac{1}{2}$ | $\frac{1}{5}$ |

#### 1.2.

A discrete random variable $X$ has five possible outcomes ($1,2,3,4,$ or $5$), and the PMF is given by:

| $x$ | $1$ | $2$ | $3$ | $4$ | $5$ |
| --- | --- | --- | --- | --- | --- |
| $P\left(X=x\right)$ | $0.25$ | $0.35$ | $0.05$ | $0.2$ | $0.1$ |

#### 1.3.

A coin is tossed, where the probability of tails is $70$ and heads is $30$. Let $X$ represent the result of the coin toss. Complete the table below:

| $x$ | Heads | Tails |
| --- | --- | --- |
| $P\left(X=x\right)$ | $0.7$ | $0.3$ |

#### 1.4.

The PMF for a random variable $X$ is given as:

| $x$ | 1 | 2 | 3 | 4 |
| --- | --- | --- | --- | --- |
| $P\left(X=x\right)$ | $1/10$ | $2/10$ | $3/10$ | $4/10$ |

## Q2

For each of the following valid random variables with associated probability density function, work out the expected value and variance.

#### 2.1.

Let $X$ be a continuous random variable on the interval $\left[0,2\right]$ with the PDF:

$$f\left(x\right)=\left\{\begin{matrix}\frac{1}{2}&if 0\leq x\leq 2\\0&otherwise\end{matrix}\right.$$

#### 2.2.

Let $X$ be a continuous random variable with the PDF:

$$f\left(x\right)=\left\{\begin{matrix}2x&if 0\leq x\leq 1,\\0&otherwise.\end{matrix}\right.$$

## Q3

Give the expected value and variance for rolling seven fair $6$-sided dice. You may assume that each roll is independent of every other roll.

## Q4

This question refers to the exponential distribution for a continuous random variable. You can find more information about this and [Factsheet: Exponential distribution](../factsheets/f-expdist.qmd).

The PDF of the exponential distribution is $P\left(X=x\right)=λe^{−λx}$. Using integration by parts (see [Guide: Integration by parts]) and the fact that

$$\lim\_{x\to \infty }x^{n}e^{−λx}=0$$

for any natural number $n$ and real $λ>0$, show that

1. the mean $μ$ of the exponential distribution is $\frac{1}{λ}$;
2. the variance $σ^{2}$ of the exponential distribution is $\frac{1}{λ^{2}}$.

[After attempting the questions above, please click this link to find the answers.](../answers/as-expectedvariance.qmd)

## Version history and licensing

v1.0: initial version created 08/25 by tdhc.

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