Questions: Law of total probability and Bayes’ theorem

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Summary

A selection of questions to test your understanding of the law of total probability and Bayes’ theorem.

*Before attempting these questions it is highly recommended that you read* [*Guide: Law of total probability and Bayes’ theorem.*](../studyguides/bayestheorem.qmd)

## Q1

Use the law of total probability to answer the following.

#### 1.1.

In a hospital:

* $40\%$ of patients are treated in Ward A,
* $60\%$ in Ward B,
* the probability of recovery within 3 days is $80\%$ in Ward A,
* the probability of recovery within 3 days is $60\%$ in Ward B.

Let $R$ be the event that a patient recovers in 3 days. What is $P\left(R\right)$?

#### 1.2.

A school has three types of lunches:

* $50\%$ of students choose vegetarian
* $30\%$ choose chicken
* $20\%$ choose fish

The probability that a student finishes their lunch is:

* $90\%$ for vegetarian
* $70\%$ for chicken
* $80\%$ for fish

What is the probability that a randomly chosen student finishes their lunch?

#### 1.3.

The magnificent Mersenne Macarons are manufactured in three Cantor’s Confectionery factories:

* $20\%$ from Factory 1 (with a defect rate $5\%$)
* $30\%$ from Factory 2 (with a defect rate $2\%$)
* $50\%$ from Factory 3 (with a defect rate $1\%$)

What is the probability that a randomly chosen Mersenne Macaron is defective?

#### 1.4.

A student can study in three locations:

* At home ($50\%$ of the time)
* In the library ($30\%$)
* In a café ($20\%$)

The probability they complete their homework is:

* $70\%$ at home
* $90\%$ in the library
* $60\%$ in the café

What is the probability that a randomly selected student completes their homework?

## Q2

Use Bayes’ theorem to answer the following.

#### 2.1.

Statistics for a test for a disease is:

* $95\%$ accurate for infected individuals (true positive)
* $90\%$ accurate for uninfected individuals (true negative)
* $2\%$ of the population has the disease

Let $D$ be the event that a person has the disease and $T$ the event they test positive. What is $P\left(D∣T\right)$?

#### 2.2.

In St Andrews, Scotland:

* $60\%$ of days are dry
* $40\%$ are rainy

A forecast predicts rain:

* $80\%$ of the time on rainy days
* $10\%$ of the time on dry days

If the forecast predicts rain in St Andrews, what is the probability that it will actually rain?

#### 2.3.

In a Cantor’s Confectionery factory:

* $70\%$ of Bayes Biscuits are made by Machine A
* $30\%$ by Machine B

The probability of a broken Bayes Biscuit is:

* $2\%$ from Machine A
* $5\%$ from Machine B

If a biscuit is broken, what is the probability it came from Machine B?

#### 2.4.

A brand new bag of Gauss Gummies contains:

* $40\%$ red sweets
* $60\%$ blue sweets

A red sweet has a $30\%$ chance of having a wrapper and a blue sweet has a $70\%$ chance of having a wrapper. If a sweet is picked at random and has a wrapper, what is the probability it is red?

[After attempting the questions above, please click this link to find the answers.](../answers/as-bayestheorem.qmd)

## Version history and licensing

v1.0: initial version created 05/25 by Sophie Chowgule as part of a University of St Andrews VIP project.

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