Proof: Rules of differentiation

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Summary

The proof sheet demonstrates that the common rules of differentiation - the sum, difference, product, quotient, and chain rules - are true.

Before reading this proof sheet, it is essential that you read Guide: Introduction to differentiation and the derivative. In addition, reading [Guide: Introduction to limits] is useful. Further reading will be illustrated where required.

The starting point of this proof sheet is the limit definition of the derivative of a function:

> **Reminder of limit definition of the derivative**
>
> The **derivative of** $f(x)$ **with respect to** x is defined to be the function
>
> $$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}.$$

Sum and difference rules

> **The sum and difference rules**
>
> (**sum rule**) The derivative of two functions $f(x)$ and $g(x)$ added together is the same as their derivatives $f'(x)$ and $g'(x)$ added together; that is, $(f+g)'(x) = f'(x) + g'(x)$ or
>
> $$\frac{\mathrm{d}}{\mathrm{d}x}(f+g) = \frac{\mathrm{d}f}{\mathrm{d}x} + \frac{\mathrm{d}g}{\mathrm{d}x}$$
>
> (**difference rule**) The derivative of one function $g(x)$ subtracted from another $f(x)$ is the same as the derivative $g'(x)$ subtracted from the derivative of $f'(x)$; that is $(f-g)'(x) = f'(x) - g'(x)$ or
>
> $$\frac{\mathrm{d}}{\mathrm{d}x}(f-g) = \frac{\mathrm{d}f}{\mathrm{d}x} - \frac{\mathrm{d}g}{\mathrm{d}x}$$

Proof of the sum rule

The strategy here is direct; put the function $(f+g)$ into the definition and pull the fraction apart to reveal the definitions of derivatives of f and g.

Let's start with $f(x)$ and $g(x)$ as two differentiable real-valued functions, with sum $(f+g)(x) = f(x) + g(x)$. Putting this into the limit definition of the derivative given above:

$$(f+g)'(x) = \lim_{h\to 0} \frac{(f+g)(x+h) - (f+g)(x)}{h}$$

Since $(f+g)(x) = f(x) + g(x)$, this becomes

$$\begin{aligned}(f+g)'(x) &= \lim_{h\to 0} \frac{f(x+h) + g(x+h) - (f(x) + g(x))}{h} \\ &= \lim_{h\to 0} \frac{f(x+h) + g(x+h) - f(x) - g(x)}{h}\end{aligned}$$

You can now split this into two fractions, one of which sets up the definition of $f'(x)$, and the other sets up the definition of $g'(x)$. So here

$$\begin{aligned}(f+g)'(x) &= \lim_{h\to 0} \frac{f(x+h) + g(x+h) - f(x) - g(x)}{h} \\ &= \lim_{h\to 0} \left(\frac{f(x+h) - f(x)}{h} + \frac{g(x+h) - g(x)}{h} \right)\end{aligned}$$

Now, use properties of limits (see [Guide: Introduction to limits]) to split the limits gives

$$\begin{aligned}(f+g)'(x) &= \lim_{h\to 0} \left(\frac{f(x+h) - f(x)}{h} + \frac{g(x+h) - g(x)}{h} \right) \\ &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} + \lim_{h\to 0} \frac{g(x+h) - g(x)}{h}\end{aligned}$$

and so, by the limit definition of the derivative

$$(f+g)'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} + \lim_{h\to 0} \frac{g(x+h) - g(x)}{h} = f'(x) + g'(x)$$

as required.

Proof of the difference rule

Let's start with $f(x)$ and $g(x)$ as two differentiable real-valued functions, with difference $(f-g)(x) = f(x)-g(x)$. Putting this into the limit definition of the derivative given above:

$$(f-g)'(x) = \lim_{h\to 0} \frac{(f-g)(x+h) - (f-g)(x)}{h}$$

Using the fact that $(f-g)(x) = f(x) - g(x)$, and taking care of the signs in expansion, gives

$$\begin{aligned}(f-g)'(x) &= \lim_{h\to 0} \frac{f(x+h) - g(x+h) - (f(x) - g(x))}{h} \\ &= \lim_{h\to 0} \frac{f(x+h) - g(x+h) - f(x) + g(x)}{h}\end{aligned}$$

You can now split this into two fractions, one of which sets up the definition of $f'(x)$, and the other sets up the definition of $g'(x)$. So here

$$\begin{aligned}(f-g)'(x) &= \lim_{h\to 0} \frac{f(x+h) - g(x+h) - f(x) + g(x)}{h} \\ &= \lim_{h\to 0} \left(\frac{f(x+h) - f(x)}{h} - \frac{g(x+h) - g(x)}{h} \right)\end{aligned}$$

Now, use properties of limits (see [Guide: Introduction to limits]) to split the limits gives

$$\begin{aligned}(f-g)'(x) &= \lim_{h\to 0} \left(\frac{f(x+h) - f(x)}{h} - \frac{g(x+h) - g(x)}{h} \right) \\ &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} - \lim_{h\to 0} \frac{g(x+h) - g(x)}{h}\end{aligned}$$

and so, by the limit definition of the derivative

$$(f-g)'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} - \lim_{h\to 0} \frac{g(x+h) - g(x)}{h} = f'(x) - g'(x)$$

as required.

Scaling rule

The scaling rule

The derivative of a function $f(x)$ multiplied by a real number c is the same as the derivative $f'(x)$ multiplied by c; that is $(cf)'(x) = cf'(x)$ or

$$\frac{\mathrm{d}}{\mathrm{d}x}(cf(x)) = c\frac{\mathrm{d}f}{\mathrm{d}x}$$

Proof of the scaling rule

This is similar to that of the sum and difference rules. Let's start with $f(x)$ as a differentiable real-valued function, with scaling $(cf)(x) = cf(x)$. Putting this into the limit definition of the derivative given above:

$$(cf)'(x) = \lim_{h\to 0} \frac{(cf)(x+h) + (cf)(x)}{h}$$

Using the fact that $(cf)(x) = cf(x)$ and factorizing out the c gives

$$\begin{aligned}(cf)'(x) &= \lim_{h\to 0} \frac{(cf)(x+h) - (cf)(x)}{h} \\ &= \lim_{h\to 0} \frac{cf(x+h) - cf(x)}{h} = \lim_{h\to 0} \frac{c(f(x+h) - f(x))}{h}\end{aligned}$$

Since the constant c does not depend on the variable in the limit h, you can use properties of limits (see [Guide: Introduction to limits]) to take the constant c out of the limit. This gives

$$\begin{aligned}(cf)'(x) &= \lim_{h\to 0} \frac{c(f(x+h) - f(x))}{h} \\ &= c \cdot \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}\end{aligned}$$

and so, by the limit definition of the derivative

$$(cf)'(x) = c \cdot \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = cf'(x)$$

as required.

Product rule

See Guide: The product rule for more about the product rule.

Here is the product rule, restated with $f(x) = u(x)$ and $g(x) = v(x)$ for visual ease in the proof that follows.

> **The product rule**
>
> Let $f(x)$ and $g(x)$ be two differentiable functions. Then the **product rule** says that
>
> $$(fg)'(x) = \frac{\mathrm{d}}{\mathrm{d}x}\left(f(x)g(x)\right) = f(x)g'(x) + f'(x)g(x)$$
>
> that is, the derivative of the product of $f(x)$ and $g(x)$ is equal to the product of $f(x)$ and the derivative of $g(x)$, plus the product of $g(x)$ and the derivative of $f(x)$.
> This can also be written as
>
> $$\frac{\mathrm{d}}{\mathrm{d}x}\left(f(x)g(x)\right) = f\frac{\mathrm{d}g}{\mathrm{d}x} + g\frac{\mathrm{d}f}{\mathrm{d}x}.$$

Proof of the product rule

Here's why the product rule works. It requires a little more thought than the proof of the sum rule and the scaling rule; you have to manufacture the definition of the derivative in one place by using a creative addition by 0.

So, let's start with $f(x)$ and $g(x)$ as two differentiable real-valued functions, with product $(fg)(x) = f(x)g(x)$. Putting this into the limit definition of the derivative given above:

$$(fg)'(x) = \lim_{h \to 0} \frac{(fg)(x+h) - (fg)(x)}{h}$$

Since $(fg)(x) = f(x)g(x)$, this becomes

$$(fg)'(x) = \lim_{h \to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

Now, there's no way of pulling this apart. You have to force the issue slightly by creatively adding 0. The way to do this is to add $-f(x+h)g(x) + f(x+h)g(x)$ into the numerator, and factorize in slightly different ways. This is fine to do, as $-f(x+h)g(x) + f(x+h)g(x) = 0$. Doing this, and factorizing to manufacture the definitions of $f'(x)$ and $g'(x)$ gives:

$$
\begin{aligned}
(fg)'(x) &= \lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h} \\
&= \lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x+h)g(x) + f(x+h)g(x) - f(x)g(x)}{h} \\
&= \lim_{h\to 0} \left(f(x+h)\frac{g(x+h)-g(x)}{h} + g(x)\frac{f(x+h)-f(x)}{h} \right)
\end{aligned}
$$

Using properties of limits, and the fact that $g(x)$ is constant as h varies to take it outside the limit gives

$$
\begin{aligned}
(fg)'(x) &= \lim_{h\to 0} \left(f(x+h)\frac{g(x+h)-g(x)}{h} + g(x)\frac{f(x+h)-f(x)}{h} \right) \\
&= \left(\lim_{h\to 0} f(x+h) \right) \left(\lim_{h\to 0} \frac{g(x+h)-g(x)}{h} \right) + g(x) \left(\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \right)
\end{aligned}
$$

Now, as h tends to 0, it follows that $f(x+h)$ tends to $f(x)$. The other two limits are the definitions of $g'(x)$ and $f'(x)$ respectively. Therefore, you can write that

$$
\begin{aligned}
(fg)'(x) &= \left(\lim_{h\to 0} f(x+h) \right) \left(\lim_{h\to 0} \frac{g(x+h)-g(x)}{h} \right) + g(x) \left(\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \right) \\
&= f(x)g'(x) + g(x)f'(x)
\end{aligned}
$$

which is the product rule.

Quotient rule

See Guide: The quotient rule for more about the quotient rule.

Here is the quotient rule, restated with $f(x) = u(x)$ and $g(x) = v(x)$ for visual ease in the proof that follows.

The quotient rule

Let $f(x)$ and $g(x)$ be two differentiable functions. Then the **quotient rule** says that

$$
\left(\frac{f}{g}\right)'(x) = \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{\left(g(x)\right)^2}
$$

that is, the derivative of $u(x)$ divided by $v(x)$ is equal to the difference of $u'(x)v(x)$ and $u(x)v'(x)$, divided by the square of the function $v(x)$.
This can also be written as

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{u(x)}{v(x)}\right) = \frac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}.$$

Proof of the quotient rule

Here's why the quotient rule works. Again, there is a step beyond algebraic manipulation where you have to manufacture the definition of the derivative in one place by using a creative addition by 0.

So, let's start with $f(x)$ and $g(x)$ as two differentiable real-valued functions (with $g(x)$ not the zero function), with quotient $(f/g)(x) = f(x)/g(x)$. Putting this into the limit definition of the derivative gives

$$\left(\frac{f}{g}\right)'(x) = \lim_{h\to 0}\frac{\left(\frac{f}{g}\right)(x+h) - \left(\frac{f}{g}\right)(x)}{h} = \lim_{h\to 0}\frac{\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}}{h}$$

You can try your best to reduce this down by cross-multiplying to get a common denominator of the numerator of the limit. Then, you can drop that denominator down to get a single fraction. Doing this:

$$\begin{aligned}\left(\frac{f}{g}\right)'(x) &= \lim_{h\to 0}\frac{\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}}{h}\\ &= \lim_{h\to 0}\frac{\frac{f(x+h)g(x)-f(x)g(x+h)}{g(x)g(x+h)}}{h}\\ &= \lim_{h\to 0}\frac{f(x+h)g(x) - f(x)g(x+h)}{hg(x)g(x+h)}\end{aligned}$$

Now, the hope is to pull this apart into two separate limits. Since you have no way of cancelling the h, you could try and manufacture the definitions of the derivatives of $f(x)$ and $g(x)$. You have to force the issue slightly by creatively adding 0; in this case, by adding $-f(x)g(x) + f(x)g(x) = 0$ to the numerator. In addition, you can use properties of limits to get rid of the $g(x)g(x+h)$ in the denominator. Doing these steps and simplifying gives:

$$\begin{aligned}
\left(\frac{f}{g}\right)'(x) &= \lim_{h\to 0}\frac{f(x+h)g(x)-f(x)g(x+h)}{hg(x)g(x+h)}\\
&= \left(\lim_{h\to 0}\frac{f(x+h)g(x)-f(x)g(x+h)}{h}\right)\left(\lim_{h\to 0}\frac{1}{g(x)g(x+h)}\right)\\
&= \left(\lim_{h\to 0}\frac{f(x+h)g(x)-f(x)g(x)+f(x)g(x)-f(x)g(x+h)}{h}\right)\left(\lim_{h\to 0}\frac{1}{g(x)g(x+h)}\right)
\end{aligned}$$

Now, factorizing this expression, using the properties of limits) and moving $g(x)$ and $-f(x)$ (notice that this needs to be done to ensure the correct definition of the derivative) out of the limits where appropriate gives

$$\begin{aligned}
\left(\tfrac{f}{g}\right)'(x) &= \left(\lim_{h\to 0}\tfrac{f(x+h)g(x)-f(x)g(x)+f(x)g(x)-f(x)g(x+h)}{h}\right)\left(\lim_{h\to 0}\tfrac{1}{g(x)g(x+h)}\right)\\
&= \left(\lim_{h\to 0}\tfrac{g(x)(f(x+h)-f(x))-f(x)(g(x+h)-g(x))}{h}\right)\left(\lim_{h\to 0}\tfrac{1}{g(x)g(x+h)}\right)\\
&= \left(\left(\lim_{h\to 0}\tfrac{g(x)(f(x+h)-f(x))}{h}\right)+\left(\lim_{h\to 0}\tfrac{-f(x)(g(x+h)-g(x))}{h}\right)\right)\left(\lim_{h\to 0}\tfrac{1}{g(x)g(x+h)}\right)\\
&= \left(g(x)\left(\lim_{h\to 0}\tfrac{f(x+h)-f(x)}{h}\right)-f(x)\left(\lim_{h\to 0}\tfrac{g(x+h)-g(x)}{h}\right)\right)\left(\lim_{h\to 0}\tfrac{1}{g(x)g(x+h)}\right)
\end{aligned}$$

Now, as h tends to 0, it follows that $g(x+h)$ tends to $g(x)$, implying that the final limit tends to $1/(g(x))^2$. The other two limits are precisely the definitions of $f'(x)$ and $g'(x)$. Therefore, you can write that

$$\begin{aligned}
\left(\frac{f}{g}\right)'(x) &= \left(g(x)\left(\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\right)-f(x)\left(\lim_{h\to 0}\frac{g(x+h)-g(x)}{h}\right)\right)\left(\lim_{h\to 0}\frac{1}{g(x)g(x+h)}\right)\\
&= (g(x)f'(x)-g'(x)f(x))\cdot\frac{1}{(g(x))^2} = \frac{f'(x)g(x)-f(x)g'(x)}{(g(x))^2}
\end{aligned}$$

which is the quotient rule.

Chain rule

See Guide: The chain rule for more about the chain rule.

Here is the chain rule, restated with $f(x) = u(x)$ and $g(x) = v(x)$ for visual ease in the proof that follows.

The chain rule

Let $f(x)$ and $g(x)$ be two differentiable functions. Then the **chain rule** says that

$$(f \circ g)'(x) = \frac{\mathrm{d}}{\mathrm{d}x}(f(g(x))) = f'(g(x)) \cdot g'(x)$$

that is, the derivative of $f(x)$ composed with $g(x)$ with respect to x is equal to the product of the derivative of f with respect to g and the derivative of g with respect to x.

This can also be written as

$$\frac{\mathrm{d}}{\mathrm{d}x}(f(g(x))) = \frac{\mathrm{d}f}{\mathrm{d}g} \cdot \frac{\mathrm{d}g}{\mathrm{d}x}.$$

Proof of the chain rule

Here's why the chain rule can be used. The idea is to take the limit definition of $(f \circ g)'(x)$ and split the limit into the product of the two derivatives $f'(g(x))$ and $g'(x)$. It requires more thought than the proofs of the product and chain rule, primarily due to the reliance on definitions of differentiation and the fact that it isn't a creative addition of 0 that splits the derivative, but a creative multiplication by 1 instead.

Alternative definition of derivative

Proving the chain rule requires the restatement of the limit definition of a derivative at a point a. Here are the two definitions side by side.

Limit definition of the derivative (1)

The **derivative of** $f(x)$ **with respect to** x **at the point** a is defined to be

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

Limit definition of the derivative (2)

The **derivative of** $f(x)$ **with respect to** x **at the point** a is defined to be

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}.$$

(See [Guide: Introduction to differentiability] for more.) To see that these are equal, start with definition (1). Here, h is the variable as the limit depends on h. Now, rescale the limit by setting $h = x - a$ (see [Guide: Properties of limits] for more). This gives

$$\begin{aligned} f'(a) &= \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} \\ &= \lim_{x-a\to 0} \frac{f(a+(x-a)) - f(a)}{x-a}. \end{aligned}$$

As $x - a \to 0$, it follows that $x \to a$; in addition, $a + (x - a) = x$. So the limit becomes

$$f'(a) = \lim_{x-a\to 0} \frac{f(a+(x-a)) - f(a)}{x-a} = \lim_{x\to a} \frac{f(x) - f(a)}{x-a}$$

so the definitions are the same at a point. Since a function f is differentiable on an interval I of real numbers if and only if $f'(a)$ exists for all a in I, it follows that you can use this definition for a differentiable function.

Intuition

The idea is to start with the second limit definition of the derivative above and put the function $(f \circ g)(x) = f(g(x))$ into the definition to get:

$$(f \circ g)'(a) = \lim_{x\to a} \frac{f(g(x)) - f(g(a))}{x-a}.$$

Now, you would want to generate the derivative of f with respect to $g(x)$ at a and the derivative of g with respect to x at a. To do this, you can notice that the $x - a$ is already there for the derivative of g. You can multiply top and bottom of the fraction by $g(x) - g(a)$. Since $\frac{(g(x)-g(a))}{(g(x)-g(a))} = 1$, this does not change the value of the limit. This gives

$$(f \circ g)'(a) = \lim_{x\to a} \left(\frac{f(g(x)) - f(g(a))}{x-a} \cdot \frac{(g(x) - g(a))}{(g(x) - g(a))} \right).$$

You can now pull this limit apart to attempt to make the two definitions of $f'(g(a))$ and $g'(a)$. Using the properties of limits to do this gives

$$\begin{aligned} (f \circ g)'(a) &= \lim_{x\to a} \left(\frac{f(g(x)) - f(g(a))}{x-a} \cdot \frac{(g(x) - g(a))}{(g(x) - g(a))} \right) \\ &= \lim_{x\to a} \frac{f(g(x)) - f(g(a))}{g(x) - g(a)} \cdot \lim_{x\to a} \frac{(g(x) - g(a))}{x-a}. \end{aligned}$$

The second of these terms is $g'(a)$, which is what you want. The first of these terms **would** be the definition of $f'(g(a))$... if the limit was $g(x) \to g(a)$ rather than $x \to a$. Here is the problem, because you **cannot guarantee** the behaviour of $g(x) - g(a)$ as x gets closer to a; it could be that $g(x) - g(a) = 0$, which is a big problem. In fact, it could be that as x gets closer to a, then $g(x) - g(a)$ could be 0 in infinitely many different places. This needs to be rectified.

Overcoming the technicality

The idea is to 'fill in' the places where $g(x) - g(a) = 0$, by defining the value of the function $\frac{f(g(x)) - f(g(a))}{g(x) - g(a)}$ at these points. You can define the function

$$\phi(y) = \begin{cases} \dfrac{f(y) - f(g(a))}{y - g(a)} & \text{if } y \neq g(a) \\ f'(g(a)) & \text{if } y = g(a) \end{cases}$$

You can notice here that $f'(g(a))$ is already defined as f is a differentiable function, meaning that $f'(y)$ exists for all y.

Now, consider the expression

$$\phi(g(x)) \cdot \frac{(g(x) - g(a))}{x - a}.$$

The idea is to prove that

$$\frac{f(g(x)) - f(g(a))}{x - a} = \phi(g(x)) \cdot \frac{(g(x) - g(a))}{x - a}$$

for all x. This way, you can evaluate the limit of the right hand side instead of the left hand side. However, this does depend on whether or not $g(x) = g(a)$.

- If $g(x) \neq g(a)$, then $g(x) - g(a) \neq 0$. You can use the first part of the definition to say that

$$\phi(g(x)) \cdot \frac{(g(x) - g(a))}{x - a} = \frac{f(g(x)) - f(g(a))}{g(x) - g(a)} \cdot \frac{(g(x) - g(a))}{x - a}$$

  Since $g(x) - g(a) \neq 0$, you can cancel these to get

$$\phi(g(x)) \cdot \frac{(g(x) - g(a))}{x - a} = \frac{f(g(x)) - f(g(a))}{x - a}.$$

- If $g(x) = g(a)$ then $g(x) - g(a) = 0$ and also $f(g(x)) - f(g(a)) = 0$. This implies that

$$\phi(g(x)) \cdot \frac{(g(x) - g(a))}{x - a} = \phi(g(x)) \cdot 0 = 0.$$

So they really are equal. Using this expression, together with the properties of limits gives

$$
\begin{aligned}
(f \circ g)'(a) &= \left(\frac{f(g(x)) - f(g(a))}{x - a} \right) \\
&= \lim_{x \to a} \phi\left(g(x)\right) \cdot \lim_{x \to a} \frac{g(x) - g(a)}{x - a}
\end{aligned}
$$

The idea is then to prove that these two limits exist; as then $(f \circ g)'(a)$ would exist. The second of these limits is precisely the definition of $g'(a)$, so let's focus on the limit of $\phi(g(x))$ as x tends to a. If this function $\phi \circ g$ is continuous at a (see [Guide: Introduction to continuity]) then this limit exists and is equal to $\phi(g(a))$. The function ϕ is defined whenever f is. Since f is differentiable, then it is continuous at every point, including $g(a)$; therefore, ϕ is continuous at $g(a)$. Since g is differentiable at a, then g is continuous at a. Therefore, by properties of continuous functions (see [Guide: Introduction to continuity]), $\phi \circ g$ is continuous at a. It follows that

$$\lim_{x \to a} \phi\left(g(x)\right) = \phi(g(a)) = f'(g(a))$$

by definition and so

$$
\begin{aligned}
(f \circ g)'(a) &= \lim_{x \to a} \phi\left(g(x)\right) \cdot \lim_{x \to a} \frac{g(x) - g(a)}{x - a} \\
&= f'(g(a)) \cdot g'(a)
\end{aligned}
$$

and this is the chain rule!

Further reading

Click this link to go back to Guide: Introduction to differentiation and the derivative.

Click this link to go back to Guide: The product rule.

Click this link to go back to Guide: The quotient rule.

Click this link to go back to Guide: The chain rule

For questions on differentiation and the derivative, please go to Questions: Introduction to differentation and the derivative.

Version history

v1.0: initial version created in 05/25 by tdhc.