Factsheet: Trigonometric identities (radians)

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Summary

A list of trigonometric identities with angles measured in radians.

*The main study guide for this factsheet is* [*Guide: Trigonometric identities (radians)*](../studyguides/trigonometricidentities-radians.qmd)*. If you would like to know more about these, please read the guide.*

*This factsheet measures angles in radians. For the associated factsheet measuring angles in degrees, please go to* [*Factsheet: Trigonometric identities (degrees)*](f-trigonometricidentities-degrees.qmd)*.*

## Trigonometric identities

**Periodicity and parity**

For all angles $θ$ and for all whole numbers $k\in Z$:

$$\begin{matrix}cos\left(−θ\right)&=cos\left(θ\right)\\sin\left(−θ\right)&=−sin\left(θ\right)\\tan\left(−θ\right)&=−tan\left(θ\right)\\cos\left(θ+2kπ\right)&=cos\left(θ\right)\\sin\left(θ+2kπ\right)&=sin\left(θ\right)\\tan\left(θ+kπ\right)&=tan\left(θ\right)\end{matrix}$$

**Pythagorean formulas**

For all angles $θ$

$$\begin{matrix}cos^{2}\left(θ\right)+sin^{2}\left(θ\right)&=1\\1+tan^{2}\left(θ\right)&=sec^{2}\left(θ\right)\\cot^{2}\left(θ\right)+1&=csc^{2}\left(θ\right)\end{matrix}$$

**Sum and difference formulas**

For all angles $α,β$:

$$\begin{matrix}cos\left(α+β\right)&=cos\left(α\right)cos\left(β\right)−sin\left(α\right)sin\left(β\right)\\cos\left(α−β\right)&=cos\left(α\right)cos\left(β\right)+sin\left(α\right)sin\left(β\right)\\sin\left(α+β\right)&=sin\left(α\right)cos\left(β\right)+cos\left(α\right)sin\left(β\right)\\sin\left(α−β\right)&=sin\left(α\right)cos\left(β\right)−cos\left(α\right)sin\left(β\right)\\tan\left(α+β\right)&=\frac{tan\left(α\right)+tan\left(β\right)}{1−tan\left(α\right)tan\left(β\right)}\\tan\left(α−β\right)&=\frac{tan\left(α\right)−tan\left(β\right)}{1+tan\left(α\right)tan\left(β\right)}\end{matrix}$$

**Double angle formulas**

For all angles $θ$:

$$\begin{matrix}cos\left(2θ\right)&=cos^{2}\left(θ\right)−sin^{2}\left(θ\right)\\sin\left(2θ\right)&=2sin\left(θ\right)cos\left(θ\right)\\tan\left(2θ\right)&=\frac{2tan\left(θ\right)}{1−tan^{2}\left(θ\right)}\end{matrix}$$

**Shift formulas**

For all angles $θ$:

$$\begin{matrix}cos\left(θ+\frac{π}{2}\right)&=−sin\left(θ\right)\\cos\left(θ−\frac{π}{2}\right)&=sin\left(θ\right)\\sin\left(θ+\frac{π}{2}\right)&=cos\left(θ\right)\\sin\left(θ−\frac{π}{2}\right)&=−cos\left(θ\right)\\cos\left(θ\pm π\right)&=−cos\left(θ\right)\\sin\left(θ\pm π\right)&=−sin\left(θ\right)\\sin\left(π−θ\right)&=sin\left(θ\right)\\cos\left(π−θ\right)&=−cos\left(θ\right)\end{matrix}$$

**Sine and cosine rules**

For a triangle with corners $A,B,C$, angles $α$, $β$, $γ$ respectively at those corners, and sides $a,b,c$ opposite their respective corners, the **sine rule** is

$$\frac{sin\left(α\right)}{a}=\frac{sin\left(β\right)}{b}=\frac{sin\left(γ\right)}{c}$$

and the **cosine rule** is

$$a^{2}=b^{2}+c^{2}−2bccos\left(α\right).$$

**Common values of trigonometric functions**

| Angle $θ$ | $0$ | $\frac{π}{6}$ | $\frac{π}{4}$ | $\frac{π}{3}$ | $\frac{π}{2}$ | $\frac{2π}{3}$ | $\frac{3π}{4}$ | $\frac{5π}{6}$ | $π$ |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| $sinθ$ | $0$ | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | $1$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ | $0$ |
| $cosθ$ | $1$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ | $0$ | $−\frac{1}{2}$ | $−\frac{\sqrt{2}}{2}$ | $−\frac{\sqrt{3}}{2}$ | $−1$ |
| $tanθ$ | $0$ | $\frac{1}{\sqrt{3}}$ | $1$ | $\sqrt{3}$ | undef. | $−\sqrt{3}$ | $−1$ | $−\frac{1}{\sqrt{3}}$ | $0$ |

## Version history

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