Factsheet: Rules of calculus

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Summary

A list of common rules in calculus.

*Please note: clickable links lead to study guides where the rule is introduced.*

## Rules of differentiation

[**Limit definition of the derivative:**](../studyguides/introtodifferentiation.qmd) If $f\left(x\right)$ is a continuous function, then (if it exists) the derivative $f′\left(x\right)$ is defined by

$$\lim\_{h\to 0}\frac{f\left(x+h\right)−f\left(x\right)}{h}$$

[**Sum/difference and constant rule:**](../studyguides/introtodifferentiation.qmd) If $f\left(x\right)$ and $g\left(x\right)$ are differentiable functions, then

$$\frac{d}{dx}\left(f\left(x\right)\pm g\left(x\right)\right)=f′\left(x\right)\pm g′\left(x\right)  and  \frac{d}{dx}\left(cf\left(x\right)\right)=c\frac{d}{dx}\left(f\left(x\right)\right)=cf′\left(x\right)$$

[**Product rule:**](../studyguides/productrule.qmd) If $f\left(x\right)=u\left(x\right)v\left(x\right)$,

$$f′\left(x\right)=\frac{d}{dx}\left(u\left(x\right)v\left(x\right)\right)=u\left(x\right)v′\left(x\right)+u′\left(x\right)v\left(x\right)$$

[**Quotient rule:**](../studyguides/quotientrule.qmd) If $f\left(x\right)=u\left(x\right)/v\left(x\right)$ and $v\left(x\right)\ne 0$, then

$$f′\left(x\right)=\frac{d}{dx}\left(\frac{u\left(x\right)}{v\left(x\right)}\right)=\frac{v\left(x\right)u′\left(x\right)−u\left(x\right)v′\left(x\right)}{\left(v\left(x\right)\right)^{2}}$$

[**Chain rule:**](../studyguides/chainrule.qmd) If $f\left(x\right)=f\left(u\left(x\right)\right)$, then

$$f′\left(x\right)=\frac{df}{du}⋅\frac{du}{dx}=f′\left(u\left(x\right)\right)⋅u′\left(x\right)$$

where $f′\left(u\left(x\right)\right)$ is the derivative of $f\left(u\right)$ with respect to $u$.

**Implicit differentiation:** If $f\left(x,y\right)=0$ defines a function $g\left(y\right)$ implicitly, then

$$\frac{d}{dx}\left(g\left(y\right)\right)=\frac{dg}{dy}⋅\frac{dy}{dx}=g′\left(y\right)⋅\frac{dy}{dx}$$

where $g′\left(y\right)$ is the derivative of $g\left(y\right)$ with respect to $y$.

## Rules of integration

**Sum/difference and constant rules**: If $f,g$ are functions and $k$ is any number:

$$∫f\left(x\right)\pm g\left(x\right) dx=∫f\left(x\right) dx\pm ∫g\left(x\right) dx  and  ∫kf\left(x\right) dx=k∫f\left(x\right) dx$$

**Limit manipulation**: If $f$ is a function and $a,b$ are real numbers, then:

* for $c$ such that $a<c<b$, then:

$$\int\_{a}^{b}f\left(x\right) dx=\int\_{a}^{c}f\left(x\right) dx+\int\_{c}^{b}f\left(x\right) dx$$

* if $a\leq b$, then:

$$\int\_{a}^{b}f\left(x\right) dx=−\int\_{b}^{a}f\left(x\right) dx$$

**Integration by substitution:** For an indefinite integral,

$$∫f\left(u\left(x\right)\right)⋅u′\left(x\right) dx=∫f\left(u\right) du$$

and for a definite integral

$$\int\_{a}^{b}f\left(u\left(x\right)\right)⋅u′\left(x\right) dx=\int\_{u\left(a\right)}^{u\left(b\right)}f\left(u\right) du$$

**Integration by parts:** For functions $u,v$ of $x$:

$$∫uv′dx=uv−∫vu′ dx$$

**Integration of derivative over function:** For a function $f$,

$$∫\frac{f′\left(x\right)}{f\left(x\right)} dx=ln\left|f\left(x\right)\right|+C.$$

## Version history

v1.0: created in 08/25 by tdhc.

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