Factsheet: Hyperbolic identities

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Summary

A list of hyperbolic trig identities.

These are common definitions and identities for hyperbolic functions. For derivatives and antiderivatives, please see Factsheet: List of derivatives and Factsheet: List of integrals respectively.

Definitions of hyperbolic functions

For all real numbers x:

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$
$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$
$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$
$$\coth(x) = \frac{1}{\tanh(x)} = \frac{\cosh(x)}{\sinh(x)} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$
$$\operatorname{sech}(x) = \frac{1}{\cosh(x)} = \frac{2}{e^x + e^{-x}}$$
$$\operatorname{csch}(x) = \frac{1}{\sinh(x)} = \frac{2}{e^x - e^{-x}}$$

Hyperbolic identities

Pythagorean formulas

For all real numbers x:

$$\cosh^2(x) - \sinh^2(x) = 1$$
$$1 - \tanh^2(x) = \operatorname{sech}^2(x)$$
$$\coth^2(x) - 1 = \operatorname{csch}^2(x)$$

Sum and difference formulas

For all real numbers x, y:

$$\cosh(x+y) = \cosh(x)\cosh(y) + \sinh(x)\sinh(y)$$

$$\cosh(x-y) = \cosh(x)\cosh(y) - \sinh(x)\sinh(y)$$

$$\sinh(x+y) = \sinh(x)\cosh(y) + \cosh(x)\sinh(y)$$

$$\sinh(x-y) = \sinh(x)\cosh(y) - \cosh(x)\sinh(y)$$

$$\tanh(x+y) = \frac{\tanh(x) + \tanh(y)}{1 + \tanh(x)\tanh(y)}$$

$$\tanh(x-y) = \frac{\tanh(x) - \tanh(y)}{1 - \tanh(x)\tanh(y)}$$

Double angle formulas

For all real numbers x:

$$\cosh(2x) = \cosh^2(x) + \sinh^2(x)$$

$$\sinh(2x) = 2\sinh(x)\cosh(x)$$

$$\tanh(2x) = \frac{2\tanh(x)}{1 + \tanh^2(x)}$$

Definitions of inverse hyperbolic functions

function	logarithmic definition	validity
$\sinh^{-1}(x)$	$\ln\left(x + \sqrt{x^2+1}\right)$	
$\cosh^{-1}(x)$	$\ln\left(x + \sqrt{x^2-1}\right)$	$x \geq 1$
$\tanh^{-1}(x)$	$\frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$	$\lvert x\rvert < 1$
$\coth^{-1}(x)$	$\frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$	$\lvert x\rvert > 1$
$\operatorname{sech}^{-1}(x)$	$\ln\left(\frac{1}{x} + \sqrt{\frac{1}{x^2} - 1}\right)$	$0 < x \leq 1$

function	logarithmic definition	validity
$\operatorname{csch}^{-1}(x)$	$\ln\left(\frac{1}{x}+\sqrt{\frac{1}{x^2}+1}\right)$	$x \neq 0$

Version history

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