Factsheet: Hyperbolic identities

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Summary

A list of hyperbolic trig identities.

*These are common definitions and identities for hyperbolic functions. For derivatives and antiderivatives, please see* [*Factsheet: List of derivatives*](f-commonderivatives.qmd) *and* [*Factsheet: List of integrals*](f-commonintegrals.qmd) *respectively.*

## Definitions of hyperbolic functions

For all real numbers $x$:

$$\begin{matrix}cosh\left(x\right)&=\frac{e^{x}+e^{−x}}{2}\\sinh\left(x\right)&=\frac{e^{x}−e^{−x}}{2}\\tanh\left(x\right)&=\frac{sinh\left(x\right)}{cosh\left(x\right)}=\frac{e^{x}−e^{−x}}{e^{x}+e^{−x}}\\coth\left(x\right)&=\frac{1}{tanh\left(x\right)}=\frac{cosh\left(x\right)}{sinh\left(x\right)}=\frac{e^{x}+e^{−x}}{e^{x}−e^{−x}}\\sech\left(x\right)&=\frac{1}{cosh\left(x\right)}=\frac{2}{e^{x}+e^{−x}}\\csch\left(x\right)&=\frac{1}{sinh\left(x\right)}=\frac{2}{e^{x}−e^{−x}}\end{matrix}$$

## Hyperbolic identities

**Pythagorean formulas**

For all real numbers $x$:

$$\begin{matrix}cosh^{2}\left(x\right)−sinh^{2}\left(x\right)&=1\\1−tanh^{2}\left(x\right)&=sech^{2}\left(x\right)\\coth^{2}\left(x\right)−1&=csch^{2}\left(x\right)\end{matrix}$$

**Sum and difference formulas**

For all real numbers $x,y$:

$$\begin{matrix}cosh\left(x+y\right)&=cosh\left(x\right)cosh\left(y\right)+sinh\left(x\right)sinh\left(y\right)\\cosh\left(x−y\right)&=cosh\left(x\right)cosh\left(y\right)−sinh\left(x\right)sinh\left(y\right)\\sinh\left(x+y\right)&=sinh\left(x\right)cosh\left(y\right)+cosh\left(x\right)sinh\left(y\right)\\sinh\left(x−y\right)&=sinh\left(x\right)cosh\left(y\right)−cosh\left(x\right)sinh\left(y\right)\\tanh\left(x+y\right)&=\frac{tanh\left(x\right)+tanh\left(y\right)}{1+tanh\left(x\right)tanh\left(y\right)}\\tanh\left(x−y\right)&=\frac{tanh\left(x\right)−tanh\left(y\right)}{1−tanh\left(x\right)tanh\left(y\right)}\end{matrix}$$

**Double angle formulas**

For all real numbers $x$:

$$\begin{matrix}cosh\left(2x\right)&=cosh^{2}\left(x\right)+sinh^{2}\left(x\right)\\sinh\left(2x\right)&=2sinh\left(x\right)cosh\left(x\right)\\tanh\left(2x\right)&=\frac{2tanh\left(x\right)}{1+tanh^{2}\left(x\right)}\end{matrix}$$

## Definitions of inverse hyperbolic functions

| function | logarithmic definition | validity |
| --- | --- | --- |
| $sinh^{−1}\left(x\right)$ | $ln\left(x+\sqrt{x^{2}+1}\right)$ |  |
| $cosh^{−1}\left(x\right)$ | $ln\left(x+\sqrt{x^{2}−1}\right)$ | $x\geq 1$ |
| $tanh^{−1}\left(x\right)$ | $\frac{1}{2}ln\left(\frac{1+x}{1−x}\right)$ | $\left|x\right|<1$ |
| $coth^{−1}\left(x\right)$ | $\frac{1}{2}ln\left(\frac{x+1}{x−1}\right)$ | $\left|x\right|>1$ |
| $sech^{−1}\left(x\right)$ | $ln\left(\frac{1}{x}+\sqrt{\frac{1}{x^{2}}−1}\right)$ | $0<x\leq 1$ |
| $csch^{−1}\left(x\right)$ | $ln\left(\frac{1}{x}+\sqrt{\frac{1}{x^{2}}+1}\right)$ | $x\ne 0$ |

## Version history

v1.0: created in 08/25 by tdhc.

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