Factsheet: Gamma distribution

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Summary

A factsheet for the gamma distribution.



An example of the gamma distribution with $α=2$ and $θ=1$.

**Where to use:** The gamma distribution generalizes the exponential distribution, allowing for greater or lesser variance. It is used to model positive continuous random variables that have skewed distributions.

**Notation:** $X∼Gamma\left(α,θ\right)$ or $X∼Gam\left(α,θ\right)$

**Parameters:** Two real numbers $α$ and $θ$, which are related to the mean $μ$ and variance $σ^{2}$:

* $α=\frac{μ^{2}}{σ^{2}}$ (shape parameter)
* $θ=\frac{σ^{2}}{μ}$ (scale parameter)

| Quantity | Value | Notes |
| --- | --- | --- |
| **Mean** | $E\left(X\right)=αθ$ |  |
| **Variance** | $V\left(X\right)=αθ^{2}$ |  |
| **PDF** | $P\left(X=x\right)=\frac{x^{α−1}exp\left(−\frac{x}{θ}\right)}{Γ\left(α\right)θ^{α}}$ | $Γ\left(x\right)$ the gamma function of $x$ |
| **CDF** | $P\left(X\leq x\right)=\frac{Gam\left(α,\frac{x}{θ}\right)}{Γ\left(α\right)}$ | $Gam\left(α,θ\right)$ is the PDF of the gamma distribution |

**Example:** You collect historical data on the time to failure of a machine from Cantor’s Confectionery. The mean is 83 days and the variance is 50.3. You can then use this to estimate the shape and scale parameters of the gamma distribution:

* $α=\frac{83^{2}}{50.3}=136.958250497≈137$
* $θ=\frac{50.3}{83}=0.60602409638≈0.61$

The distribution can be expressed as $X∼Gam\left(137,0.61\right)$, where the shape parameter is 137 and the scale parameter is 0.61.

# Further reading

[This interactive element appears in Overview: Probability distributions. Please click this link to go to the guide.](../overviews/o-distributions.qmd)

## Version history

v1.0: initial version created 04/25 by tdhc and Michelle Arnetta as part of a University of St Andrews VIP project.

* v1.1: moved to factsheet form and populated with material from [Overview: Probability distributions](../overviews/o-distributions.qmd) by tdhc.

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