Factsheet: List of integrals

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Summary

A list of common (and some uncommon) integrals of functions.

Throughout, $a,k$ are real numbers and $C$ is the constant of integration.

## Antiderivatives of polynomial, rational, exponential, logarithmic functions

| function | antiderivative w.r.t $x$ | notes |
| --- | --- | --- |
| $a$ | $ax+C$ |  |
| $ax^{n}$ | $\frac{ax^{n+1}}{n+1}+C$ | $n\in R, n\ne −1$ |
| $ax^{−1}$ | $aln\left|x\right|+C$ |  |
| $\frac{a}{bx+c}$ | $\frac{a}{b}ln\left|bx+c\right|+C$ | $b,c\in R$ |
| $\frac{a}{\left(bx+c\right)^{n}}$ | $\frac{a\left(bx+c\right)^{1−n}}{b\left(1−n\right)}+C$ | $b,c\in R$ |
| $ae^{kx}$ | $\frac{a}{k}e^{kx}+C$ |  |
| $aln\left(kx\right)$ | $axln\left|kx\right|−ax+C$ |  |

## Antiderivatives of trigonometric functions

| function | antiderivative w.r.t $x$ |
| --- | --- |
| $asin\left(kx\right)$ | $−\frac{a}{k}cos\left(kx\right)+C$ |
| $acos\left(kx\right)$ | $\frac{a}{k}sin\left(kx\right)+C$ |
| $atan\left(kx\right)$ | $\frac{a}{k}ln\left|sec\left(kx\right)\right|+C$ |
| $acot\left(kx\right)$ | $\frac{a}{k}ln\left|sin\left(kx\right)\right|+C$ |
| $asec\left(kx\right)$ | $\frac{a}{k}ln\left|tan\left(kx\right)+sec\left(kx\right)\right|+C$ |
| $acsc\left(kx\right)$ | $\frac{a}{k}\left(ln\left|sin\left(\frac{kx}{2}\right)\right|−ln\left|cos\left(\frac{kx}{2}\right)\right|\right)+C$ |

## Antiderivatives of some hyperbolic functions

| function | antiderivative w.r.t $x$ |
| --- | --- |
| $asinh\left(kx\right)$ | $\frac{a}{k}cosh\left(kx\right)+C$ |
| $acosh\left(kx\right)$ | $\frac{a}{k}sinh\left(kx\right)+C$ |
| $atanh\left(kx\right)$ | $\frac{a}{k}ln\left|cosh\left(kx\right)\right|+C$ |
| $acoth\left(kx\right)$ | $\frac{a}{k}ln\left|sinh\left(kx\right)\right|+C$ |

## Standard forms that integrate to inverse trigonometric/hyperbolic functions

| function | antiderivative w.r.t $x$ |
| --- | --- |
| $\frac{a}{\sqrt{1−k^{2}x^{2}}}$ | $\frac{a}{k}sin^{−1}\left(kx\right)+C$ |
| $−\frac{a}{\sqrt{1−k^{2}x^{2}}}$ | $\frac{a}{k}cos^{−1}\left(kx\right)+C$ |
| $\frac{a}{1+k^{2}x^{2}}$ | $\frac{a}{k}tan^{−1}\left(kx\right)+C$ |
| $\frac{a}{\sqrt{1+k^{2}x^{2}}}$ | $\frac{a}{k}sinh^{−1}\left(kx\right)+C$ |
| $\frac{a}{\sqrt{k^{2}x^{2}−1}}$ | $\frac{a}{k}cosh^{−1}\left(kx\right)+C$ |
| $\frac{a}{\sqrt{1−k^{2}x^{2}}}$ | $\frac{a}{k}tanh^{−1}\left(kx\right)+C$ |

# Further reading

For more about where these came from, please see [Guide: Introduction to integration](../studyguides/introtointegration.qmd) and [Proof sheet: Antiderivatives of other common functions].

## Version history

v1.0: created in 08/25 by tdhc.

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