Factsheet: Chi-squared distribution

Michelle Arnetta and Tom Coleman

Summary

A factsheet for the χ^2 distribution.

Chi-Squared(k = 5)

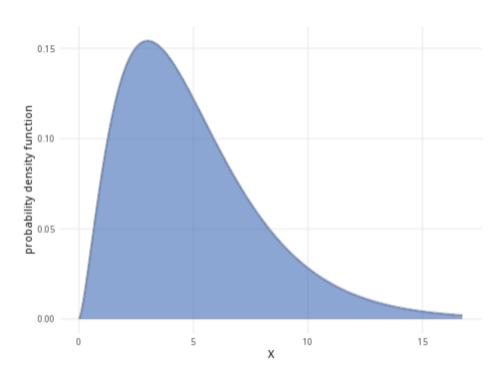


Figure 1: An example of the chi-squared distribution with k=5.

Where to use: The χ^2 distribution is used for hypothesis testing, such as for goodness of fit tests and tests for independence. (See Guide: Introduction to hypothesis testing for more.) It is a special case of the gamma distribution, as $\chi^2(k) = \operatorname{Gam}(\frac{k}{2},2)$.

Notation: $X \sim \chi^2(k)$

Parameter: The integer k is the number of degrees of freedom in the sample.

Quantity	Value	Notes
Mean	$\mathbb{E}(X) = k$	

Quantity	Value	Notes
Variance	$\mathbb{V}(X) = 2k$	
PDF	$\mathbb{P}(X = x) = \frac{x^{\frac{k}{2} - 1} \exp\left(-\frac{x}{2}\right)}{2^{\frac{k}{2}} \Gamma\left(\frac{k}{2}\right)}$ $\mathbb{P}(X \le x) = \frac{1}{\Gamma\left(\frac{k}{2}\right)} \operatorname{Gam}\left(\frac{k}{2}, \frac{x}{2}\right)$	$\Gamma(\boldsymbol{x})$ is the gamma function
CDF	$\mathbb{P}(X \le x) = \frac{1}{\Gamma\left(\frac{k}{2}\right)} \operatorname{Gam}\left(\frac{k}{2}, \frac{x}{2}\right)$	$\Gamma(x)$ is the gamma function, $\operatorname{Gam}(\alpha,\theta)$ is the PDF of the gamma distribution

Examples:

• Goodness of fit example: You have a six-sided die with six possible outcomes: 1, 2, 3, 4, 5, and 6. You calculate the expected frequencies of each outcome. Then you roll the die many times and record the observed frequencies of each outcome. Since there are 6 categories,

degrees of freedom = number of categories
$$-1 = 6 - 1 = 5$$

This can be expressed as $X \sim \chi^2(5)$, meaning the degrees of freedom is 5.

■ Test for independence example: You are investigating whether there is a correlation between two variables: candy colour and flavour. You have 5 categories of colours and 3 categories of flavours. Calculating the degrees of freedom can be done with the formula:

$$(\mathsf{categories} \ \mathsf{of} \ \mathsf{colours} - 1)(\mathsf{categories} \ \mathsf{of} \ \mathsf{flavours} - 1) = (5-1)(3-1) = (4)(2) = 8.$$

You can model $X \sim \chi^2(8)$, meaning that there are 8 degrees of freedom.

Further reading

This interactive element appears in Overview: Probability distributions. Please click this link to go to the guide.

Version history

v1.0: initial version created 04/25 by tdhc and Michelle Arnetta as part of a University of St Andrews VIP project.

• v1.1: moved to factsheet form and populated with material from Overview: Probability distributions by tdhc.

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