Answers: Trigonometric identities (radians)

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Summary

A selection of questions on trigonometric identities, using radians to measure angles.

*These are the answers to* [*Questions: Trigonometric identities (radians)*](../questions/qs-trigonometricidentities-radians.qmd)*.*

**Please attempt the questions before reading these answers!**

## Q1

1.1. $ 2\left(6sin^{2}\left(θ\right)\right)+3\left(4cos^{2}\left(θ\right)\right)=12\left(sin^{2}\left(θ\right)+cos^{2}\left(θ\right)\right)=12$

1.2. $ 10\left(7sin^{2}\left(θ\right)\right)+14\left(5cos^{2}\left(θ\right)\right)=70$

1.3. $ 5\left(\frac{6}{csc^{2}\left(θ\right)}\right)+15\left(\frac{2}{sec^{2}\left(θ\right)}\right)=30$

1.4. $ \left(cos^{2}\left(θ\right)−sin^{2}\left(θ\right)\right)^{2}+4sin^{2}\left(θ\right)cos^{2}\left(θ\right)=cos^{2}\left(2θ\right)+sin^{2}\left(2θ\right)=1$

1.5. $ 2sin\left(π/6\right)cos\left(π/12\right)+2cos\left(π/6\right)sin\left(π/12\right)=2sin\left(π/6+π/12\right)=2sin\left(π/4\right)=\sqrt{2}$

1.6. $ 3cos\left(π/4\right)cos\left(π/12\right)−3sin\left(π/4\right)sin\left(π/12\right)=3cos\left(π/3\right)=\frac{3}{2}$

1.7. $ sin\left(5π/6\right)+sin\left(π/6\right)=2sin\left(\frac{180}{2}\right)cos\left(\frac{120}{2}\right)=2sin\left(π/2\right)cos\left(π/3\right)=1$

1.8. $ cos\left(5π/6\right)+sin\left(π/6\right)=2cos\left(π/2\right)cos\left(π/3\right)=0$

## Q2

2.1. $ tan\left(θ\right)cos\left(−θ\right)=\frac{sin\left(θ\right)}{cos\left(θ\right)}⋅cos\left(θ\right)=sin\left(θ\right)$

2.2 $ tan\left(−θ\right)csc\left(−θ\right)sec\left(−θ\right)=\left(−\frac{sin\left(θ\right)}{cos\left(θ\right)}\right)\left(\frac{1}{−sin\left(θ\right)}\right)\left(\frac{1}{cos\left(θ\right)}\right)=\left(\frac{1}{cos^{2}\left(θ\right)}\right)=sec^{2}\left(θ\right)$

2.3. $ tan^{2}\left(θ\right)+sin^{2}\left(θ\right)+cos^{2}\left(θ\right)=tan^{2}\left(θ\right)+1=sec^{2}\left(θ\right)$

2.4. $ \frac{2sin\left(θ\right)}{cos\left(θ\right)\left(1−tan^{2}\left(θ\right)\right)}=tan\left(2θ\right)$

2.5. $ \frac{sin\left(7θ\right)+sin\left(3θ\right)}{cos\left(7θ\right)−cos\left(3θ\right)}=\frac{2sin\left(5θ\right)cos\left(2θ\right)}{−2sin\left(5θ\right)sin\left(2θ\right)}=−cot\left(θ\right)$

2.6. $ \frac{sin\left(5θ\right)−sin\left(θ\right)}{cos\left(5θ\right)+cos\left(θ\right)}=tan\left(2θ\right)$

## Q3

3.1. $cos\left(5π/6\right)=\frac{\sqrt{3}}{2}$

3.2. Here $sin\left(3π/4\right)=\frac{1}{\sqrt{2}}$, and $sin\left(5π/4\right)=−\frac{1}{\sqrt{2}}$.

3.3. $cos\left(13π/18\right)=−0.766$ to three decimal places.

## Q4

4.1. $sin\left(π/12\right)=sin\left(π/4\right)cos\left(π/6\right)−cos\left(π/4\right)sin\left(π/6\right)=\frac{\sqrt{3}}{2\sqrt{2}}−\frac{1}{2\sqrt{2}}=\frac{\sqrt{3}−1}{2\sqrt{2}}$

4.2. $ cos\left(π/12\right)=\frac{\sqrt{3}+1}{2\sqrt{2}}$

4.3. $ tan\left(π/12\right)=\frac{\sqrt{3}+1}{\sqrt{3}−1}$

4.4. $ sin\left(5π/12\right)=sin\left(π/4\right)cos\left(π/6\right)+cos\left(π/4\right)sin\left(π/6\right)=\frac{\sqrt{3}+1}{2\sqrt{2}}$

4.5. $ cos\left(5π/12\right)=\frac{\sqrt{3}−1}{2\sqrt{2}}$

4.6. $ tan\left(5π/12\right)=\frac{\sqrt{3}+1}{\sqrt{3}−1}$

## Version history and licensing

v1.0: initial version created 08/23 by Dzhemma Ruseva as part of a University of St Andrews STEP project.

* v1.1: edited 05/24 by tdhc, and split into versions for both degrees and radians.

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