Answers: Solving equations involving logarithms

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Summary

Answers to questions relating to solving equations involving logarithms.

These are the answers to Questions: Solving equations involving logarithms

Please attempt the questions before reading these answers!

Throughout this answer sheet, the natural logarithm $\log_e(x)$ is written as $\ln(x)$.

Q1

1.1. Here, $y = x^{1/4}$ and $x = y^4$.

1.2. Here, $y = x^3$ and $x = y^{1/3}$.

1.3. Here, $x = \sqrt{y^{1/2}x^3}$, $y = x^4/z^6$, $z = \sqrt[3]{x^2y^{-1/2}}$.

Q2

2.1. The solution to $6\log_3(x) + \log_3(5) = 9$ is $x = \sqrt[6]{\frac{3^9}{5}}$.

2.2. The solution to $\log_2(16x) = 6$ is $x = 4$.

2.3. The solution to $\log_{12} e^{2t} = 4$ is $t = 2\ln(12) = \ln(144)$.

2.4. The solution to $log_9(x) + log_3(3x) = 6$ is $x = 3^{10/3}$.

2.5. The solution to $4\ln\sqrt{x} - \ln(1-2x) = 0$ is $x = -1 + \sqrt{2}$.

2.6. The solution to $\ln(x+1) - \ln(x) = e$ is $x = \dfrac{1}{e^e - 1}$.

2.7. There are no solutions to $\log_{10}(2y+10) = \log_{10}(y-2)$.

2.8. The solutions to $\log_3\sqrt{x} - \log_9\sqrt{4x-3} = 0$ are $x = 1$ and $x = 3$.

2.9. The solutions to $\log_3(2-3x) = \log_9(6x^2-19x+2)$ are $x = -1/3$ and $x = -2$.

2.10. The solutions to $\log_3(x) - 2\log_x(3) = 1$ are $x = 9$ and $x = 1/3$.

Q3

The solutions are $x = 15$ and $y = 1/2$.

Version history and licensing

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