Answers: The scalar product

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Summary

Answers to questions relating to the guide on the scalar product.

*These are the answers to* [*Questions: The scalar product*](../questions/qs-scalarproduct.qmd)*.*

**Please attempt the questions before reading these answers!**

## Q1

1.1. For $a=\left(\begin{matrix}6\\3\\4\end{matrix}\right)$ and $b=\left(\begin{matrix}1\\4\\2\end{matrix}\right)$, the scalar product is $a⋅b=26$.

1.2. For $a=\left(\begin{matrix}10\\−7\\4\end{matrix}\right)$ and $b=\left(\begin{matrix}3\\−5\\13\end{matrix}\right)$, the scalar product is $a⋅b=117$.

1.3. For $a=\left(\begin{matrix}−44\\−12\\3\end{matrix}\right)$ and $b=\left(\begin{matrix}61\\−25\\93\end{matrix}\right)$, the scalar product is $a⋅b=−2237$.

1.4. For $a=\left(\begin{matrix}54\\38\\0\end{matrix}\right)$ and $b=\left(\begin{matrix}32\\−55\\13\end{matrix}\right)$, the scalar product is $a⋅b=−362$.

1.5. For $a=2i+7j+k$ and $b=6i+4j+8k$, the scalar product is $a⋅b=48$.

1.6. For $a=−3i+10j−8k$ and $b=i−12j+9k$, the scalar product is $a⋅b=−195$.

1.7. For $a=17j+23k$ and $b=6i−23j−8k$, the scalar product is $a⋅b=−575$.

1.8. For $a=i$ and $b=j$, the scalar product is $a⋅b=0$.

As the scalar product of $a=i$ and $b=j$ is $0$, they are perpendicular to each other. This is true for any combination of any *distinct* pair of $i$, $j$, and $k$. However, since any vector is parallel to itself, it follows that $i⋅i=\left|i\right|\left|i\right|=\left|1\right|\left|1\right|=1$; similar results hold for $j⋅j$ and $k⋅k$.

## Q2

2.1. For $a=\left(\begin{matrix}−5\\2\\−3\end{matrix}\right)$ and $b=\left(\begin{matrix}2\\−2\\11\end{matrix}\right)$, the angle $θ$ is $132.2^{∘}$.

2.2. For $a=\left(\begin{matrix}1\\1\\1\end{matrix}\right)$ and $b=\left(\begin{matrix}1\\−1\\1\end{matrix}\right)$, the angle $θ$ is $70.5^{∘}$.

2.3. For $a=\left(\begin{matrix}−8\\1\\−4\end{matrix}\right)$ and $b=\left(\begin{matrix}−1\\−5\\7\end{matrix}\right)$, the angle $θ$ is $108.7^{∘}$.

2.4. For $a=\left(\begin{matrix}1.2\\−1.4\\−3.1\end{matrix}\right)$ and $b=\left(\begin{matrix}−5.4\\9.7\\−7.5\end{matrix}\right)$, the angle $θ$ is $86.2^{∘}$.

2.5. For $a=\left(\begin{matrix}45\\65\\54\end{matrix}\right)$ and $b=\left(\begin{matrix}−19\\−58\\71\end{matrix}\right)$, the angle $θ$ is $95.1^{∘}$.

2.6. For $a=\left(\begin{matrix}1\\0\\0\end{matrix}\right)$ and $b=\left(\begin{matrix}0\\0\\1\end{matrix}\right)$, the angle $θ$ is $90^{∘}$.

2.7. For $a=\left(\begin{matrix}−1\\−2\\3\end{matrix}\right)$ and $b=\left(\begin{matrix}4\\−5\\6\end{matrix}\right)$, the angle $θ$ is $43.0^{∘}$.

2.8. For $a=\left(\begin{matrix}−17\\3\\8\end{matrix}\right)$ and $b=\left(\begin{matrix}12\\−19\\−16\end{matrix}\right)$, the angle $θ$ is $137.8^{∘}$.

## Q3

3.1. For $a=\left(\begin{matrix}2\\4\\7\end{matrix}\right)$ and $b=\left(\begin{matrix}1\\λ\\−2\end{matrix}\right)$ to be perpendicular, then $λ=3$.

3.2. For $a=\left(\begin{matrix}0\\1\\λ\end{matrix}\right)$ and $b=\left(\begin{matrix}1\\2\\3\end{matrix}\right)$ to be perpendicular, then $λ=−\frac{2}{3}$.

3.3. For $a=\left(\begin{matrix}9\\−2\\11\end{matrix}\right)$ and $b=\left(\begin{matrix}λ\\−λ\\3\end{matrix}\right)$ to be perpendicular, then $λ=−3$.

3.4. For $a=\left(\begin{matrix}λ\\6\\1\end{matrix}\right)$ and $b=\left(\begin{matrix}λ\\λ\\8\end{matrix}\right)$ to be perpendicular, then $λ=−2$ or $λ=−4$.

3.5. For $a=\left(\begin{matrix}−2λ^{2}\\4\\14\end{matrix}\right)$ and $b=\left(\begin{matrix}3\\2λ\\1\end{matrix}\right)$ to be perpendicular, then $λ=\frac{7}{3}$ or $λ=−1$.

3.6. For $a=\left(\begin{matrix}−5\\9\\2λ\end{matrix}\right)$ and $b=\left(\begin{matrix}λ\\−2\\λ\end{matrix}\right)$ to be perpendicular, then $λ=\frac{9}{2}$ or $λ=−2$.

3.7. For $a=\left(\begin{matrix}−7\\4\\2λ\end{matrix}\right)$ and $b=\left(\begin{matrix}2λ\\1\\6λ\end{matrix}\right)$ to be perpendicular, then $λ=\frac{2}{3}$ or $λ=\frac{1}{2}$.

3.8. For $a=\left(\begin{matrix}−25\\−1λ^{2}\\−2\end{matrix}\right)$ and $b=\left(\begin{matrix}3λ\\−11\\7\end{matrix}\right)$ to be perpendicular, then $λ=7$ or $λ=−\frac{2}{11}$.

## Version history and licensing

v1.0: initial version created 08/23 by Ritwik Anand as part of a University of St Andrews STEP project.

* v1.1: edited 05/24 by tdhc.

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