Answers: Rationalizing the denominator

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Summary

Answers to questions relating to the guide on rationalizing the denominator.

*These are the answers to* [*Questions: Rationalizing the denominator*](../questions/qs-rationalizingthedenominator.qmd)*.*

**Please attempt the questions before reading these answers!**

## Q1

1.1. $ \frac{5}{\sqrt{3}}=\frac{5\sqrt{3}}{3}$

1.2. $ \frac{7}{2\sqrt{5}}=\frac{7\sqrt{5}}{10}$

1.3. $ \frac{11}{4\sqrt{7}}=\frac{11\sqrt{7}}{28}$

1.4. $ \frac{8}{5\sqrt{6}}=\frac{4\sqrt{6}}{15}$

1.5. $ \frac{3\sqrt{2}}{\sqrt{5}}=\frac{3\sqrt{10}}{5}$

1.6. $ \frac{9}{\sqrt{10}}=\frac{9\sqrt{10}}{10}$

1.7. $ \frac{\sqrt{7}}{\sqrt{3}}=\frac{\sqrt{21}}{3}$

1.8. $ \frac{\sqrt{2}}{\sqrt{6}}=\frac{\sqrt{3}}{3}$

1.9. $ \frac{12}{\sqrt{11}}=\frac{12\sqrt{11}}{11}$

1.10. $ \frac{\sqrt{8}}{\sqrt{2}}=2$

1.11. $ \frac{15}{3\sqrt{7}}=\frac{5\sqrt{7}}{7}$

1.12. $ \frac{6\sqrt{3}}{\sqrt{10}}=\frac{3\sqrt{30}}{5}$

1.13. $ \frac{\sqrt{18}}{\sqrt{9}}=\sqrt{2}$

1.14. $ \frac{2\sqrt{5}}{\sqrt{12}}=\frac{\sqrt{30}}{3}$

1.15. $ \frac{4}{\sqrt{2}}=2\sqrt{2}$

1.16. $ \frac{10}{5\sqrt{13}}=\frac{2\sqrt{13}}{13}$

## Q2

2.1. $ \frac{5}{2+\sqrt{3}}=10−5\sqrt{3}$

2.2. $ \frac{7}{4−\sqrt{2}}=\frac{4+\sqrt{2}}{2}$

2.3. $ \frac{3}{\sqrt{5}+1}=\frac{3\sqrt{5}−3}{4}$

2.4. $ \frac{\sqrt{7}}{\sqrt{3}−1}=\frac{\sqrt{21}+\sqrt{7}}{2}$

2.5. $ \frac{2+\sqrt{5}}{1−\sqrt{2}}=−2−2\sqrt{2}−\sqrt{5}−\sqrt{10}$

2.6. $ \frac{3\sqrt{2}+5}{4+\sqrt{6}}=\frac{12\sqrt{2}−6\sqrt{3}+20−5\sqrt{6}}{10}$

2.7. $ \frac{8}{3−\sqrt{7}}=12+4\sqrt{7}$

2.8. $ \frac{6}{2+\sqrt{5}}=−12+6\sqrt{5}$

2.9. $ \frac{\sqrt{10}}{\sqrt{2}+3}=\frac{3\sqrt{10}−2\sqrt{5}}{7}$

2.10. $ \frac{2\sqrt{3}+5}{\sqrt{7}−1}=\frac{2\sqrt{21}+5\sqrt{7}+2\sqrt{3}+5}{6}$

2.11. $ \frac{\sqrt{6}−\sqrt{2}}{2+\sqrt{5}}=−2\sqrt{6}+2\sqrt{5}+2\sqrt{2}−\sqrt{10}$

2.12. $ \frac{4+\sqrt{3}}{5−\sqrt{7}}=\frac{4\sqrt{7}+5\sqrt{3}+\sqrt{21}+20}{18}$

2.13. $ \frac{2}{4−\sqrt{11}}=\frac{8+2\sqrt{11}}{5}$

2.14. $ \frac{\sqrt{8}+\sqrt{3}}{\sqrt{7}−2}=\frac{2\sqrt{14}+4\sqrt{2}+\sqrt{21}+2\sqrt{3}}{3}$

## Q3

3.1. $ $ To prove this equation, rationalize the denominator of the left hand side of the equation.

Since the denominator contains two square roots you can multiply the numerator and denominator by $−2\sqrt{3}+\sqrt{5}$ or by $2\sqrt{3}−\sqrt{5}$ to rationalize the denominator.

If you multiply the numerator and denominator by $2\sqrt{3}−\sqrt{5}$ you get:

$$\frac{\sqrt{11}}{2\sqrt{3}+\sqrt{5}}⋅\frac{2\sqrt{3}−\sqrt{5}}{2\sqrt{3}−\sqrt{5}}=\frac{\sqrt{11}\left(2\sqrt{3}−\sqrt{5}\right)}{\left(2\sqrt{3}+\sqrt{5}\right)\left(2\sqrt{3}−\sqrt{5}\right)}$$

Expanding the brackets in both the numerator and the denominator gives you:

$$\frac{\sqrt{11}\left(2\sqrt{3}−\sqrt{5}\right)}{\left(2\sqrt{3}+\sqrt{5}\right)\left(2\sqrt{3}−\sqrt{5}\right)}=\frac{2\sqrt{33}−\sqrt{55}}{\left(2\sqrt{3}\right)^{2}−2\sqrt{15}+2\sqrt{15}−\left(\sqrt{5}\right)^{2}}$$

Simplifying the denominator then gives you:

$$\frac{2\sqrt{33}−\sqrt{55}}{\left(2\sqrt{3}\right)^{2}−2\sqrt{15}+2\sqrt{15}−\left(\sqrt{5}\right)^{2}}=\frac{2\sqrt{33}−\sqrt{55}}{4\left(3\right)−5}$$

Simplifying further gives you the final answer and the right hand side of the equation you are proving:

$$\frac{2\sqrt{33}−\sqrt{55}}{4\left(3\right)−5}=\frac{2\sqrt{33}−\sqrt{55}}{7}$$

If you instead multiply the numerator and denominator by $−2\sqrt{3}+\sqrt{5}$ you get:

$$\frac{\sqrt{11}}{2\sqrt{3}+\sqrt{5}}⋅\frac{−2\sqrt{3}+\sqrt{5}}{−2\sqrt{3}+\sqrt{5}}=\frac{\sqrt{11}\left(−2\sqrt{3}+\sqrt{5}\right)}{\left(2\sqrt{3}+\sqrt{5}\right)\left(−2\sqrt{3}+\sqrt{5}\right)}$$

Expanding the brackets in both the numerator and the denominator gives you:

$$\frac{\sqrt{11}\left(−2\sqrt{3}+\sqrt{5}\right)}{\left(2\sqrt{3}+\sqrt{5}\right)\left(−2\sqrt{3}+\sqrt{5}\right)}=\frac{\sqrt{11}\left(−2\sqrt{3}+\sqrt{5}\right)}{−\left(2\sqrt{3}\right)^{2}+2\sqrt{15}−2\sqrt{15}+\left(\sqrt{5}\right)^{2}}$$

Simplifying the denominator gives you:

$$\frac{\sqrt{11}\left(−2\sqrt{3}+\sqrt{5}\right)}{−\left(2\sqrt{3}\right)^{2}+2\sqrt{15}−2\sqrt{15}+\left(\sqrt{5}\right)^{2}}=\frac{−2\sqrt{33}+\sqrt{55}}{5−4\left(3\right)}$$

Further simplifying the denominator then gives you:

$$\frac{−2\sqrt{33}+\sqrt{55}}{5−4\left(3\right)}=\frac{−2\sqrt{33}+\sqrt{55}}{−7}$$

To get a positive denominator, multiplying both the numerator and the denominator by $−1$ gives you the right hand side of the equation you are proving:

$$\frac{−2\sqrt{33}+\sqrt{55}}{−7}=\frac{2\sqrt{33}−\sqrt{55}}{7}$$

3.2. $ \frac{5−\sqrt{2}}{\sqrt{10}−\sqrt{3}}=\frac{5\sqrt{10}+5\sqrt{3}−2\sqrt{5}−\sqrt{6}}{7}$

## Version history and licensing

v1.0: initial version created 12/24 by Maximilian Volmar.

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