Answers: The product rule

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Summary

Answers to questions relating to the guide on the product rule.

These are the answers to Questions: The product rule.

Please attempt the questions before reading these answers!

1.1. $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(xe^x\right) = e^x + xe^x.$

1.2. $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(x^2e^{2x}\right) = 2xe^{2x} + 2x^2e^{2x}.$

1.3. As $\tan(x) = \sin(x)/cos(x)$, the function becomes $5x^3\sin(x)$ and so

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(5x^3\tan(x)\cos(x)\right) = \frac{\mathrm{d}}{\mathrm{d}x}\left(5x^3\sin(x)\right) = 15x^2\sin(x) + 5x^3\cos(x).$$

1.4. $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(x\ln(x)\right) = \ln(x) + 1.$

1.5. $\dfrac{\mathrm{d}}{\mathrm{d}x}\left((x^3+x^2-5)(x+1)\right) = (3x^2+2x)(x+1) + (x^3+x^2-5).$

1.6. $\dfrac{\mathrm{d}}{\mathrm{d}x}\left((13x^2+5x+2)(x^3+2)\right) = (26x+5)(x^3+2) + 3x^2(13x^2+5x+2).$

1.7. Take the x inside the first bracket so the function becomes $(5x^3+3x^2+2x)(x^2+x+1)$. Then

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(x(5x^2+3x+2)(x^2+x+1)\right) = \frac{\mathrm{d}}{\mathrm{d}x}\left((5x^3+3x^2+2x)(x^2+x+1)\right)$$

$$= (10x^2+6x+2)(x^2+x+1) + (5x^3+3x^2+2x)(2x+1).$$

1.8. $$\frac{\mathrm{d}}{\mathrm{d}x}\left((10x^2+21)\cos(x)\right) = 20x\cos(x) - (10x^2+21)\sin(x).$$

1.9. Using the definitions of $\cosh(2x)$ and $\sinh(3x)$:

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\cosh(2x)\sinh(3x)\right) = \frac{\mathrm{d}}{\mathrm{d}x}\left(\left(\frac{e^{2x}+e^{-2x}}{2}\right)\left(\frac{e^{3x}-e^{-3x}}{2}\right)\right)$$

$$= \left(\frac{e^{2x}+e^{-2x}}{2}\right)\left(\frac{3e^{3x}+3e^{-3x}}{2}\right) + \left(\frac{2e^{2x}-2e^{-2x}}{2}\right)\left(\frac{e^{3x}-e^{-3x}}{2}\right)$$

$$= 3\cosh(2x)\cosh(3x) + 2\sinh(2x)\sinh(3x)$$

1.10. $$\frac{\mathrm{d}}{\mathrm{d}x}\left((x^2+3)\ln(x)\right) = 2x\ln(x) + \frac{x^2+3}{x}.$$

1.11. $$\frac{\mathrm{d}}{\mathrm{d}x}\left(\sin(x)\sqrt{x}\right) = \cos(x)\sqrt{x} + \frac{\sin(x)}{2\sqrt{x}}.$$

1.12. Since $\cosh(x) = \dfrac{e^x+e^{-x}}{2}$, it follows that

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\cosh(x)\ln(x)\right) = \frac{\mathrm{d}}{\mathrm{d}x}\left(\left(\frac{e^x+e^{-x}}{2}\right)\ln(x)\right)$$

$$= \left(\frac{e^x-e^{-x}}{2}\right)\ln(x) + \left(\frac{e^x+e^{-x}}{2}\right)\frac{1}{x}$$

$$= \sinh(x)\ln(x) + \frac{\cosh(x)}{x}$$

since $\sinh(x) = \dfrac{e^x-e^{-x}}{2}$.

1.13. Factorize to get $x^2(\sqrt{x}+\cos(x))$, then

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(x^2\sqrt{x}+x^2\cos(x)\right)=\frac{\mathrm{d}}{\mathrm{d}x}\left(x^2(\sqrt{x}+\cos(x))\right)$$
$$=2x(\sqrt{x}+\cos(x))+x^2\left(\frac{1}{2\sqrt{x}}-\sin(x)\right).$$

1.14. $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(e^{-5x}(x^3+5)\right)=3x^2e^{-5x}-5e^{-5x}(x^3+5).$

1.15. $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{2}{5}\sinh(x)+\dfrac{2}{13}\cosh(x)\right)=\dfrac{2}{5}\cosh(x)+\dfrac{2}{13}\sinh(x).$

1.16. Using the product rule twice here:

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln(x)\ln(3x)\ln(100x)\right)=\frac{\ln(3x)\ln(100x)+\ln(x)\ln(100x)+\ln(x)\ln(3x)}{x}.$$

1.17. $\dfrac{\mathrm{d}}{\mathrm{d}x}\left((x^2+5x+2)\sin(x)\right)=(2x+5)\sin(x)+(x^2+5x+2)\cos(x).$

1.18. $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(-\ln(x)\ln(3x)\right)=-\dfrac{1}{x}(\ln(3x)+\ln(x)).$

1.19. Using the product rule twice:

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((x^5+3)(x^2+3x)(x^7+x^4)\right)$$
$$=5x^4(x^2+3x)(x^7+x^4)+(x^5+3)(2x+3)(x^7+x^4)+(x^5+3)(x^2+3x)(7x^6+4x^3).$$

1.20. $\dfrac{\mathrm{d}}{\mathrm{d}x}\left((\sin(x)+3x)e^{-x}\right)=(\cos(x)+3)e^{-x}-(\sin(x)+3x)e^{-x}.$

Version history and licensing

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