Answers: The product rule

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Summary

Answers to questions relating to the guide on the product rule.

*These are the answers to* [*Questions: The product rule*](../questions/qs-productrule.qmd)*.*

**Please attempt the questions before reading these answers!**

1.1. $ \frac{d}{dx}\left(xe^{x}\right)=e^{x}+xe^{x}.$

1.2. $ \frac{d}{dx}\left(x^{2}e^{2x}\right)=2xe^{2x}+2x^{2}e^{2x}.$

1.3. As $tan\left(x\right)=sin\left(x\right)/cos\left(x\right)$, the function becomes $5x^{3}sin\left(x\right)$ and so

$$\frac{d}{dx}\left(5x^{3}tan\left(x\right)cos\left(x\right)\right)=\frac{d}{dx}\left(5x^{3}sin\left(x\right)\right)=15x^{2}sin\left(x\right)+5x^{3}cos\left(x\right).$$

1.4. $ \frac{d}{dx}\left(xln\left(x\right)\right)=ln\left(x\right)+1.$

1.5. $ \frac{d}{dx}\left(\left(x^{3}+x^{2}−5\right)\left(x+1\right)\right)=\left(3x^{2}+2x\right)\left(x+1\right)+\left(x^{3}+x^{2}−5\right).$

1.6. $ \frac{d}{dx}\left(\left(13x^{2}+5x+2\right)\left(x^{3}+2\right)\right)=\left(26x+5\right)\left(x^{3}+2\right)+3x^{2}\left(13x^{2}+5x+2\right).$

1.7. Take the $x$ inside the first bracket so the function becomes $\left(5x^{3}+3x^{2}+2x\right)\left(x^{2}+x+1\right)$. Then

$$\begin{matrix}\frac{d}{dx}\left(x\left(5x^{2}+3x+2\right)\left(x^{2}+x+1\right)\right)&=\frac{d}{dx}\left(\left(5x^{3}+3x^{2}+2x\right)\left(x^{2}+x+1\right)\right)\\&=\left(10x^{2}+6x+2\right)\left(x^{2}+x+1\right)+\left(5x^{3}+3x^{2}+2x\right)\left(2x+1\right).\end{matrix}$$

1.8. $ \frac{d}{dx}\left(\left(10x^{2}+21\right)cos\left(x\right)\right)=20xcos\left(x\right)−\left(10x^{2}+21\right)sin\left(x\right).$

1.9. Using the definitions of $cosh\left(2x\right)$ and $sinh\left(3x\right)$:

$$\begin{matrix}\frac{d}{dx}\left(cosh\left(2x\right)sinh\left(3x\right)\right)&=\frac{d}{dx}\left(\left(\frac{e^{2x}+e^{−2x}}{2}\right)\left(\frac{e^{3x}−e^{−3x}}{2}\right)\right)\\&=\left(\frac{e^{2x}+e^{−2x}}{2}\right)\left(\frac{3e^{3x}+3e^{−3x}}{2}\right)+\left(\frac{2e^{2x}−2e^{−2x}}{2}\right)\left(\frac{e^{3x}−e^{−3x}}{2}\right)\\&=3cosh\left(2x\right)cosh\left(3x\right)+2sinh\left(2x\right)sinh\left(3x\right)\end{matrix}$$

1.10. $ \frac{d}{dx}\left(\left(x^{2}+3\right)ln\left(x\right)\right)=2xln\left(x\right)+\frac{x^{2}+3}{x}.$

1.11. $ \frac{d}{dx}\left(sin\left(x\right)\sqrt{x}\right)=cos\left(x\right)\sqrt{x}+\frac{sin\left(x\right)}{2\sqrt{x}}.$

1.12. Since $cosh\left(x\right)=\frac{e^{x}+e^{−x}}{2}$, it follows that

$$\begin{matrix}\frac{d}{dx}\left(cosh\left(x\right)ln\left(x\right)\right)&=\frac{d}{dx}\left(\left(\frac{e^{x}+e^{−x}}{2}\right)ln\left(x\right)\right)\\&=\left(\frac{e^{x}−e^{−x}}{2}\right)ln\left(x\right)+\left(\frac{e^{x}+e^{−x}}{2}\right)\frac{1}{x}\\&=sinh\left(x\right)ln\left(x\right)+\frac{cosh\left(x\right)}{x}\end{matrix}$$

since $sinh\left(x\right)=\frac{e^{x}−e^{−x}}{2}$.

1.13. Factorize to get $x^{2}\left(\sqrt{x}+cos\left(x\right)\right)$, then

$$\begin{matrix}\frac{d}{dx}\left(x^{2}\sqrt{x}+x^{2}cos\left(x\right)\right)&=\frac{d}{dx}\left(x^{2}\left(\sqrt{x}+cos\left(x\right)\right)\right)\\&=2x\left(\sqrt{x}+cos\left(x\right)\right)+x^{2}\left(\frac{1}{2\sqrt{x}}−sin\left(x\right)\right).\end{matrix}$$

1.14. $ \frac{d}{dx}\left(e^{−5x}\left(x^{3}+5\right)\right)=3x^{2}e^{−5x}−5e^{−5x}\left(x^{3}+5\right).$

1.15. $ \frac{d}{dx}\left(\frac{2}{5}sinh\left(x\right)+\frac{2}{13}cosh\left(x\right)\right)=\frac{2}{5}cosh\left(x\right)+\frac{2}{13}sinh\left(x\right).$

1.16. Using the product rule twice here:

$$\frac{d}{dx}\left(ln\left(x\right)ln\left(3x\right)ln\left(100x\right)\right)=\frac{ln\left(3x\right)ln\left(100x\right)+ln\left(x\right)ln\left(100x\right)+ln\left(x\right)ln\left(3x\right)}{x}.$$

1.17. $ \frac{d}{dx}\left(\left(x^{2}+5x+2\right)sin\left(x\right)\right)=\left(2x+5\right)sin\left(x\right)+\left(x^{2}+5x+2\right)cos\left(x\right).$

1.18. $ \frac{d}{dx}\left(−ln\left(x\right)ln\left(3x\right)\right)=−\frac{1}{x}\left(ln\left(3x\right)+ln\left(x\right)\right).$

1.19. Using the product rule twice:

$$\begin{matrix}\frac{d}{dx}&\left(\left(x^{5}+3\right)\left(x^{2}+3x\right)\left(x^{7}+x^{4}\right)\right)\\&=5x^{4}\left(x^{2}+3x\right)\left(x^{7}+x^{4}\right)+\left(x^{5}+3\right)\left(2x+3\right)\left(x^{7}+x^{4}\right)+\left(x^{5}+3\right)\left(x^{2}+3x\right)\left(7x^{6}+4x^{3}\right).\end{matrix}$$

1.20. $ \frac{d}{dx}\left(\left(sin\left(x\right)+3x\right)e^{−x}\right)=\left(cos\left(x\right)+3\right)e^{−x}−\left(sin\left(x\right)+3x\right)e^{−x}.$

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v1.0: initial version created 05/25 by Sara Delgado Garcia as part of a University of St Andrews VIP project.

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