Answers: PMFs, PDFs, and CDFs

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Summary

Answers to questions relating to the guide on PMFs, PDFs, and CDFs.

*These are the answers to* [*Questions: PMFs, PDFs, and CDFs.*](../questions/qs-pmfspdfscdfs.qmd)

**Please attempt the questions before reading these answers!**

## Q1

#### 1.1.

The given PMF is valid because:

**Non-negativity**: All $P\left(X=x\right)\geq 0$

**Honesty**: The sum of all probabilities equals 1:

$$\sum\_{x=1}^{4}p\left(x\right)=\sum\_{x=1}^{4}P\left(X=x\right)=\frac{1}{10}+\frac{1}{5}+\frac{1}{2}+\frac{1}{5}=1$$

$P\left(X=4\right)=\frac{1}{5}$.

#### 1.2.

The given PMF is valid because:

**Non-negativity**: All $P\left(X=x\right)\geq 0$

**Honesty**: The sum of all probabilities equals 1:

$$\sum\_{x=1}^{4}p\left(x\right)=\sum\_{x=1}^{4}P\left(X=x\right)=0.25+0.35+0.05+0.2+0.1=1$$

$P\left(X=3 or X=4\right)=0.05+0.2=0.25$

#### 1.3.

The completed PMF table for the biased coin toss is:

| $x$ | Heads | Tails |
| --- | --- | --- |
| $P\left(X=x\right)$ | 0.3 | 0.7 |

This is a valid PMF because:

**Non-negativity**: Both $P\left(X=x\right)\geq 0$

**Honesty**: The sum of both probabilities equal 1:

$$\sum\_{x}^{​}p\left(x\right)=\sum\_{x}^{​}P\left(X=x\right)=0.3+0.7=1$$

#### 1.4. {-}

This is not a valid PMF since it fails the honesty condition:

**Honesty**: The sum of the given probabilities does not equal 1:

$$\sum\_{x=1}^{7}p\left(x\right)=\sum\_{x=1}^{7}P\left(X=x\right)=0.1+0.05+0.05+0.3+0.25+0.75+0.35=1.85\ne 1$$

#### 1.5.

1. $P\left(Blue\right)=\frac{3}{10}=0.3$
2. The PMF for the given scenario is:

| $x$ | Red | Blue | Green |
| --- | --- | --- | --- |
| $P\left(X=x\right)$ | 0.5 | 0.3 | 0.2 |

This is a valid PMF because:

**Non-negativity**: All $P\left(X=x\right)\geq 0$

**Honesty**: The sum of all three probabilities equals to 1:

$$\sum\_{x}^{​}p\left(x\right)=\sum\_{x}^{​}P\left(X=x\right)=0.5+0.3+0.2=1$$

#### 1.6.

1. For the given PMF to be valid, you must have $p=\frac{1}{10}$.
2. For $p=\frac{1}{10}$, then $P\left(X=3\right)=\frac{3}{10}$.

## Q2

#### 2.1.

This is a valid PDF because:

**Non-negativity**: $f\left(x\right)\geq 0$ for all values of $x$.

**Honesty**: $\int\_{−\infty }^{\infty }f\left(x\right) dx=\int\_{0}^{2}\frac{1}{2} dx=\left[ \frac{x}{2} \right]\_{0}^{2}=1$

$P\left(1\leq x\leq 2\right)=\int\_{1}^{2}\frac{1}{2} dx=\left[ \frac{x}{2} \right]\_{1}^{2}=\frac{1}{2}$

#### 2.2.

This is a valid PDF because:

**Non-negativity**: $f\left(x\right)\geq 0$ for all values of $x$

**Honesty**: $\int\_{−\infty }^{\infty }f\left(x\right) dx=\int\_{0}^{1}\frac{x}{2} dx=\left[ x^{2} \right]\_{0}^{1}=1$

1. $P\left(0.5\leq X\leq 1\right)=\int\_{0.5}^{1}2x dx=\left[x^{2}\right]\_{0.5}^{1}=1^{2}−\left(0.5\right)^{2}=1−0.25=0.75$
2. $P\left(0.25\leq X\leq 0.75\right)=\int\_{0.25}^{0.75}2x dx=\left[x^{2}\right]\_{0.25}^{0.75}=\left(0.75\right)^{2}−\left(0.25\right)^{2}=0.5625−0.0625=0.5$

#### 2.3.

This is a valid PDF because:

**Non-negativity**: $f\left(x\right)\geq 0$ for all values of $x$

**Honesty**: $\int\_{−\infty }^{\infty }f\left(x\right) dx=\int\_{3}^{7}\frac{1}{4} dx=\left[\frac{x}{4}\right]\_{3}^{7}=1$

$P\left(3\leq X\leq 6\right)=\int\_{3}^{6}\frac{1}{4} dx=\left[\frac{x}{4}\right]\_{3}^{6}=\frac{6}{4}−\frac{3}{4}=\frac{3}{4}$

#### 2.4.

This is not a valid PDF since it does not meet the honesty condition:

**Honesty**: $\int\_{−\infty }^{\infty }f\left(x\right) dx=\int\_{1}^{4}\frac{1}{9} dx+\int\_{5}^{7}\frac{1}{4} dx\ne 1$

Calculating the individual integrals:

* $\int\_{1}^{4}\frac{1}{9} dx=\frac{1}{9}\left[x\right]\_{1}^{4}=\frac{1}{3}$
* $\int\_{5}^{7}\frac{1}{4} dx=\frac{1}{4}\left[x\right]\_{5}^{7}=\frac{1}{2}$

And adding them together:

$\int\_{−\infty }^{\infty }f\left(x\right) dx=\frac{1}{3}+\frac{1}{2}=\frac{5}{6}\ne 1$

#### 2.5.

1. For the given PDF to be valid, you must have $k=3$.
2. $P\left(0.2\leq X\leq 0.3\right)=\int\_{0.2}^{0.3}3x^{2} dx=3\left[\frac{x^{3}}{3}\right]\_{0.2}^{0.3}=\left[x^{3}\right]\_{0.2}^{0.3}=0.019$

#### 2.6.

This is a valid PDF because:

**Non-negativity**: $f\left(x\right)\geq 0$ for all values of $x$

**Honesty**: $\int\_{−\infty }^{\infty }f\left(x\right) dx=\int\_{0}^{0.5}4x dx+\int\_{0.5}^{0.75}\left(4−4x\right) dx+\int\_{0.75}^{1}0.5 dx$

Calculating the individual integrals:

* $\int\_{0}^{0.5}4x dx=\left[2x^{2}\right]\_{0}^{0.5}=0.5$
* $\int\_{0.5}^{0.75}\left(4−4x\right) dx=\left[4x−2x^{2}\right]\_{0.5}^{0.75}=0.375$
* $\int\_{0.75}^{1}0.5 dx=\left[0.5x\right]\_{0.75}^{1}=0.125$

and adding them together gives $0.5+0.375+0.125=1$.

## Q3

#### 3.1.

1. $F\left(3\right)=P\left(X\leq 3\right)=0.1+0.3+0.5=0.9$
2. $P\left(X>2\right)=1−P\left(X\leq 2\right)=1−\left(0.1+0.3+0.5\right)=1−0.9=0.1$

#### 3.2.

1. The CDF for values $0.5$, $1$, and $2$:
	* $F\left(0.5\right)=\int\_{0}^{0.5}\frac{1}{2} dx=\left[\frac{x}{2}\right]\_{0}^{0.5}=\frac{0.5}{2}=0.25$
	* $F\left(1\right)=\int\_{0}^{1}\frac{1}{2} dx=\left[\frac{x}{2}\right]\_{0}^{1}=\frac{1}{2}=0.5$
	* $F\left(2\right)=\int\_{0}^{2}\frac{1}{2} dx=\left[\frac{x}{2}\right]\_{0}^{2}=\frac{2}{2}=1$
2. $F\left(3\right)=1$ (since the CDF for any $x\geq 2$ is $1$.)

#### 3.3.

1. The CDF at points $4$, $5$, and $6$:
	* $F\left(4\right)=\int\_{3}^{4}\frac{1}{4} dx=\left[\frac{x}{4}\right]\_{3}^{4}=\frac{4}{4}−\frac{3}{4}=\frac{1}{4}$
	* $F\left(5\right)=\int\_{3}^{5}\frac{1}{4} dx=\left[\frac{x}{4}\right]\_{3}^{5}=\frac{5}{4}−\frac{3}{4}=\frac{2}{4}=\frac{1}{2}$
	* $F\left(6\right)=\int\_{3}^{6}\frac{1}{4} dx=\left[\frac{x}{4}\right]\_{3}^{6}=\frac{6}{4}−\frac{3}{4}=\frac{3}{4}$
2. $P\left(X>5\right)=1−F\left(5\right)=1−\frac{1}{2}=\frac{1}{2}$.

#### 3.4.

This is not a valid CDF because the CDF should be non-decreasing as $x$ increases.

## Version history and licensing

v1.0: initial version created 12/24 by Sophie Chowgule as part of a University of St Andrews VIP project.

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