Answers: Logarithms

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Summary

Answers to questions relating to the study guide on logarithms.

*These are answers to:* [*Questions: Logarithms*](../questions/qs-logarithms.qmd)*.*

**Please attempt the questions before reading these answers!**

Throughout this answer sheet, the natural logarithm $log\_{e}\left(x\right)$ is written as $ln\left(x\right)$.

## Q1

1.1. $ log\_{7}\left(x\right)=1$ rearranged gives $7^{1}=x$ so $x=7$.

1.2. $ log\_{8}\left(x\right)=3$ rearranged gives $8^{3}=x$ so $x=512$.

1.3. $ log\_{12}\left(x\right)=0$ rearranged gives $12^{0}=x$ so $x=1$.

1.4. $ log\_{10}\left(100\right)=x$ rearranged gives $10^{x}=100$ so $x=2$.

1.5. $ log\_{2}\left(64\right)=x$ rearranged gives $2^{x}=64$ so $x=6$.

1.6. $ log\_{4}\left(2\right)=x$ rearranged gives $4^{x}=2$ so $x=\frac{1}{2}$.

1.7. $ log\_{3}\left(27\right)=x$ rearranged gives $3^{x}=27$ so $x=3$.

1.8. $ log\_{10}\left(1\right)=x$ rearranged gives $10^{x}=1$ so $x=0$.

1.9. $ log\_{x}\left(16\right)=4$ rearranged gives $x^{4}=16$ so $x=\sqrt[4]{16}=2$.

1.10. $ log\_{x}\left(49\right)=2$ rearranged gives $x^{2}=49$ so $x=\sqrt{49}=7$.

1.11. $ log\_{x}\left(13\right)=4$ rearranged gives $x^{4}=13$ so $x=\sqrt[4]{13}$.

1.12. $ log\_{2x}\left(12\right)=−1$ rearranged gives $\left(2x\right)^{−1}=12$ so $x=\frac{1}{24}$.

## Q2

The product rule: $log\_{a}\left(M⋅N\right)=log\_{a}\left(M\right)+log\_{a}\left(N\right)$

The quotient rule: $log\_{a}\left(\frac{M}{N}\right)=log\_{a}\left(M\right)−log\_{a}\left(N\right)$

The power rule: $log\_{a}\left(M^{k}\right)=k⋅log\_{a}\left(M\right)$

The zero rule: $log\_{a}\left(1\right)=0$

The identity rule: $log\_{a}\left(a\right)=1$

2.1. The solution to $log\_{3}\left(\frac{1}{27}\right)=x$ is $x=−1/3$.

2.2. The solution to $4log\_{4}\left(2\right)=x$ is $x=2$.

2.3. The solution to $log\_{5}\left(10\right)+log\_{5}\left(\frac{5}{2}\right)=x$ is $x=2$.

2.4. The solution to $3log\_{7}\left(a^{1/3}\right)−\frac{1}{2}log\_{7}\left(a^{2}\right)=x$ is $x=0$.

2.5. The solution to $log\_{x}\left(YZ\right)=M$ is $x=\sqrt[M]{YZ}$.

2.6. The solution to $log\_{a}\left(y\right)−log\_{a}\left(x\right)=11$ is $x=ya^{−11}$.

## Q3

3.1. $log\_{3}\left(25\right)$ is equal to $\frac{2}{log\_{5}\left(3\right)}$.

3.2. $log\_{8}\left(3\right)$ is equal to $\frac{4log\_{16}\left(3\right)}{3}$.

3.3. $log\_{e}\left(10\right)$ is equal to $\frac{1}{log\_{1000}\left(e^{3}\right)}$.

3.4. $ln\left(27\right)$ is equal to $\frac{3}{log\_{3}\left(e\right)}$.

3.5. $log\_{4}\left(8x\right)$ is equal to $\frac{3}{2}+log\_{2}\left(\sqrt{2}\right)$.

## Version history and licensing

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* v1.1: edited 05/24 by tdhc.

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