Answers: Introduction to quadratic equations

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Summary

Answers to questions relating to the guide on introduction to quadratic equations.

*These are the answers to* [*Questions: Introduction to quadratic equations*](../questions/qs-introtoquadratics.qmd)*.*

**Please attempt the questions before reading these answers!**

## Q1

For each of the quadratic equations below, identify the variable and the coefficients $a,b,c$.

1.1. For $x^{2}−7x+6=0$, the variable is $x$, and the coefficients are $a=1,b=−7,c=6$.

1.2. For $y^{2}+14y+49=0$, the variable is $y$, and the coefficients are $a=1,b=14,c=49$.

1.3. For $h^{2}−h−56=0$, the variable is $h$, and the coefficients are $a=1,b=−1,c=−56$.

1.4. For $7y^{4}−y^{2}=0$, the variable is $y^{2}$, and the coefficients are $a=7,b=−1,c=0$.

1.5. For $5n^{2}−14n+100=0$, the variable is $n$, and the coefficients are $a=5,b=−14,c=100$.

1.6. For $A^{2}−144=0$, the variable is $A$, and the coefficients are $a=1,b=0,c=−144$.

1.7. For $25M^{2}=0$, the variable is $M$, and the coefficients are $a=25,b=0,c=0$.

1.8. For $e^{2x}−4e^{x}+4=0$, the variable is $e^{x}$, and the coefficients are $a=1,b=−4,c=4$.

1.9. For $−9s^{4}+3s^{2}−1=0$, the variable is $s^{2}$, and the coefficients are $a=−9,b=3,c=−1$.

1.10. For $2e^{6x}+e^{3x}+1=0$, the variable is $e^{3x}$, and the coefficients are $a=2,b=1,c=1$.

1.11. For $cos^{2}\left(x\right)+4cos\left(x\right)−4=0$, the variable is $cos\left(x\right)$, and the coefficients are $a=1,b=4,c=−4$.

1.12. For $8x^{8}−4x^{4}−1=0$, the variable is $x^{4}$, and the coefficients are $a=8,b=−4,c=−1$.

## Q2

2.1. The discriminant of the equation $x^{2}−7x+6=0$ is $D=25$, and therefore the equation has two distinct real roots.

2.2. The discriminant of the equation $y^{2}+14y+49=0$ is $D=0$, and therefore the equation has one distinct real root.

2.3. The discriminant of the equation $h^{2}−h−56=0$ is $D=217$, and therefore the equation has two distinct real roots.

2.4. The discriminant of the equation $7y^{4}−y^{2}=0$ is $D=1$, and therefore the equation has two distinct real roots.

2.5. The discriminant of the equation $5n^{2}−14n+100=0$ is $D=−1804$, and therefore the equation has no real roots (two distinct complex roots).

2.6. The discriminant of the equation $A^{2}−144=0$ is $D=576$, and therefore the equation has two distinct real roots.

2.7. The discriminant of the equation $25M^{2}=0$ is $D=0$, and therefore the equation has one distinct real root.

2.8. The discriminant of the equation $e^{2x}−4e^{x}+4=0$ is $D=0$, and therefore the equation has one distinct real root $r$ in $e^{x}$. Whether or not it has a real root in $x$ depends on whether or not $r$ is positive. If $r$ is positive, there is exactly one real root $x=ln\left(r\right)$; if $r$ is negative, then there are no real roots.

2.9. The discriminant of the equation $−9s^{4}+3s^{2}−1=0$ is $D=−27$, and therefore the equation has no real roots. This is true even with $s^{2}$ as the variable, as if $s^{2}$ is complex then $s$ must also be complex.

2.10. The discriminant of the equation $2e^{6x}+e^{3x}+1=0$ is $D=−7$, and therefore the equation has no real roots. This is true even with $e^{3x}$ as the variable, as if $e^{3x}$ is complex then $x$ must also be complex.

2.11. The discriminant of the equation $cos^{2}\left(x\right)+4cos\left(x\right)−4=0$ is $D=32$, and therefore the equation has two distinct real roots $r\_{1}$ and $r\_{2}$ in $cos\left(x\right)$. Whether or not it has a real root in $x$ depends on whether or not either of the roots is between $−1$ and $1$. If both $r\_{1}$ and $r\_{2}$ are outside this range, then there are no real roots. If one of $r\_{1}$ or $r\_{2}$ is between $−1$ and $1$, then there are infinitely many solutions.

2.12. The discriminant of the equation $8x^{8}−4x^{4}−1=0$ is $D=48$, and therefore the equation has two distinct real roots $r\_{1}$ and $r\_{2}$ in $x^{4}$. The amount of real roots depend on the signs of $r\_{1}$ and $r\_{2}$.

* If $r\_{1}$ and $r\_{2}$ are both positive, then there are four real roots in $x$. This is because $x^{2}=\pm \sqrt{r\_{1}}$ or $x^{2}=\pm \sqrt{r\_{2}}$; square rooting the positive terms gives the roots in $x$ as $\pm \sqrt{\left(\sqrt{r\_{1}}\right)}=\pm \sqrt[4]{r\_{1}}$ and $\pm \sqrt{\left(\sqrt{r\_{2}}\right)}=\pm \sqrt[4]{r\_{2}}$. Any other roots must be complex, since you are taking square roots of the negative numbers $−\sqrt{r\_{1}}$ and $−\sqrt{r\_{2}}$.
* If exactly one of $r\_{1}$ and $r\_{2}$ is positive (say $r\_{i}$), then there are two real roots in $x$ given by $\pm \sqrt[4]{r\_{i}}$. All other roots are complex.
* If both $r\_{1}$ and $r\_{2}$ are negative, then then there are no real roots in $x$.

## Q3

3.1. Rearranging gives $x^{2}+x−1=0$. The discriminant of this is $D=5$, and therefore the equation has two distinct real roots.

3.2. Rearranging gives $y^{2}+10=0$. The discriminant of this is $D=−40$, and therefore the equation has no real roots (two distinct complex roots).

3.3. Rearranging gives $4m^{2}+4m+1=0$. The discriminant of this is $D=0$, and therefore the equation has one distinct real root.

3.4. Rearranging gives $t^{4}+1=0$. The discriminant of this is $D=−4$, and therefore the equation has no real roots. This is true even with $t^{2}$ as the variable, as if $t^{2}$ is complex then $t$ must also be complex.

3.5. Rearranging gives $5x^{2}−11x−1=0$. The discriminant of this is $D=101$, and therefore the equation has two distinct real roots.

3.6. Rearranging gives $e^{2x}−2e^{x}+1=0$. The discriminant of this is $D=0$, and therefore the equation has one distinct real root $r$ in $e^{x}$. Whether or not it has a real root in $x$ depends on whether or not $r$ is positive. If $r$ is positive, there is exactly one real root $x=ln\left(r\right)$; if $r$ is negative, then there are no real roots.

## Version history and licensing

v1.0: initial version created 04/23 by tdhc.

* v1.1: edited 05/24 by tdhc.

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