Answers: Introduction to partial differentiation

Donald Campbell

Summary

Answers to questions relating to the guide on the introduction to partial differentiation.

*These are the answers to* [*Questions: Introduction to partial differentiation*](../questions/qs-introtopartialdifferentiation.qmd)*.*

**Please attempt the questions before reading these answers!**

## Q1

1.1. $ \frac{∂f}{∂x}=2xy$ and $\frac{∂f}{∂y}=x^{2}+3y^{2}$.

1.2. $ \frac{∂f}{∂x}=9x^{2}+y$ and $\frac{∂f}{∂y}=x−8y^{3}$.

1.3. $ \frac{∂f}{∂x}=2ycos\left(2x\right)$ and $\frac{∂f}{∂y}=sin\left(2x\right)$.

1.4. $ \frac{∂f}{∂x}=ye^{xy}+4xy^{3}$ and $\frac{∂f}{∂y}=xe^{xy}+6x^{2}y^{2}$.

1.5. $ \frac{∂f}{∂x}=\frac{1}{x}+ln\left(y\right)+3$ and $\frac{∂f}{∂y}=\frac{x}{y}$.

1.6. $ \frac{∂f}{∂x}=−\frac{y}{x^{2}}−\frac{1}{y}$ and $\frac{∂f}{∂y}=\frac{1}{x}+\frac{x}{y^{2}}$.

1.7. $ \frac{∂f}{∂x}=exp\left(y^{2}\right)$ and $\frac{∂f}{∂y}=2xyexp\left(y^{2}\right)$.

1.8. $ \frac{∂f}{∂x}=\frac{x}{\sqrt{x^{2}+y^{2}}}$ and $\frac{∂f}{∂y}=\frac{y}{\sqrt{x^{2}+y^{2}}}$.

1.9. $ \frac{∂f}{∂x}=12\left(3x+2y\right)^{3}$ and $\frac{∂f}{∂y}=8\left(3x+2y\right)^{3}$.

1.10. $ \frac{∂f}{∂x}=y^{2}cos\left(xy\right)$ and $\frac{∂f}{∂y}=xcos\left(xy\right)−x^{2}ysin\left(xy\right)$.

1.11. $ \frac{∂f}{∂x}=2xcos\left(x^{2}+y^{2}\right)$ and $\frac{∂f}{∂y}=2ycos\left(x^{2}+y^{2}\right)$.

1.12. $ \frac{∂f}{∂x}=\frac{2xy^{2}}{1+x^{2}y^{2}}$ and $\frac{∂f}{∂y}=\frac{2x^{2}y}{1+x^{2}y^{2}}$.

1.13. $ \frac{∂f}{∂x}=2xysin\left(z\right)$ and $\frac{∂f}{∂y}=x^{2}sin\left(z\right)$ and $\frac{∂f}{∂z}=x^{2}ycos\left(z\right)$.

1.14. $ \frac{∂f}{∂x}=\left(y+z\right)\left(2x+y+z\right)$ and $\frac{∂f}{∂y}=\left(x+z\right)\left(x+2y+z\right)$ and $\frac{∂f}{∂z}=\left(x+y\right)\left(x+y+2z\right)$.

1.15. $ \frac{∂f}{∂x}=\frac{yz\left(y+z\right)}{\left(x+y+z\right)^{2}}$ and $\frac{∂f}{∂y}=\frac{xz\left(x+z\right)}{\left(x+y+z\right)^{2}}$ and $\frac{∂f}{∂z}=\frac{xy\left(x+y\right)}{\left(x+y+z\right)^{2}}$.

## Q2

2.1. $ \frac{∂^{2}f}{∂x^{2}}=2$ and $\frac{∂^{2}f}{∂y^{2}}=−2$ so $\frac{∂^{2}f}{∂x^{2}}+\frac{∂^{2}f}{∂y^{2}}=0$.

2.2. $ \frac{∂^{2}f}{∂x^{2}}=0$ and $\frac{∂^{2}f}{∂y^{2}}=0$ so $\frac{∂^{2}f}{∂x^{2}}+\frac{∂^{2}f}{∂y^{2}}=0$.

2.3. $ \frac{∂^{2}f}{∂x^{2}}=6x$ and $\frac{∂^{2}f}{∂y^{2}}=−6x$ so $\frac{∂^{2}f}{∂x^{2}}+\frac{∂^{2}f}{∂y^{2}}=0$.

2.4. $ \frac{∂^{2}f}{∂x^{2}}=−cos\left(x\right)sinh\left(y\right)$ and $\frac{∂^{2}f}{∂y^{2}}=cos\left(x\right)sinh\left(y\right)$ so $\frac{∂^{2}f}{∂x^{2}}+\frac{∂^{2}f}{∂y^{2}}=0$.

2.5. $ \frac{∂^{2}f}{∂x^{2}}=e^{x}sin\left(y\right)$ and $\frac{∂^{2}f}{∂y^{2}}=−e^{x}sin\left(y\right)$ so $\frac{∂^{2}f}{∂x^{2}}+\frac{∂^{2}f}{∂y^{2}}=0$.

2.6. $ \frac{∂^{2}f}{∂x^{2}}=\frac{2xy}{\left(x^{2}+y^{2}\right)^{2}}$ and $\frac{∂^{2}f}{∂y^{2}}=−\frac{2xy}{\left(x^{2}+y^{2}\right)^{2}}$ so $\frac{∂^{2}f}{∂x^{2}}+\frac{∂^{2}f}{∂y^{2}}=0$.

2.7. $ \frac{∂^{2}f}{∂x^{2}}=\frac{2\left(y^{2}−x^{2}\right)}{\left(x^{2}+y^{2}\right)^{2}}$ and $\frac{∂^{2}f}{∂y^{2}}=\frac{2\left(x^{2}−y^{2}\right)}{\left(x^{2}+y^{2}\right)^{2}}$ so $\frac{∂^{2}f}{∂x^{2}}+\frac{∂^{2}f}{∂y^{2}}=0$.

## Q3

3.1. $ \frac{∂^{2}f}{∂x∂y}=2x+2y$ and $\frac{∂^{2}f}{∂y∂x}=2x+2y$ so $\frac{∂^{2}f}{∂x∂y}=\frac{∂^{2}f}{∂y∂x}$.

3.2. $ \frac{∂^{2}f}{∂x∂y}=−4xsin\left(y\right)$ and $\frac{∂^{2}f}{∂y∂x}=−4xsin\left(y\right)$ so $\frac{∂^{2}f}{∂x∂y}=\frac{∂^{2}f}{∂y∂x}$.

3.3. $ \frac{∂^{2}f}{∂x∂y}=20\left(x+y\right)^{3}$ and $\frac{∂^{2}f}{∂y∂x}=20\left(x+y\right)^{3}$ so $\frac{∂^{2}f}{∂x∂y}=\frac{∂^{2}f}{∂y∂x}$.

3.4. $ \frac{∂^{2}f}{∂x∂y}=−\frac{1}{\left(y+1\right)^{2}}$ and $\frac{∂^{2}f}{∂y∂x}=−\frac{1}{\left(y+1\right)^{2}}$ so $\frac{∂^{2}f}{∂x∂y}=\frac{∂^{2}f}{∂y∂x}$.

3.5. $ \frac{∂^{2}f}{∂x∂y}=−\frac{xy}{\left(x^{2}+y^{2}\right)^{3/2}}$ and $\frac{∂^{2}f}{∂y∂x}=−\frac{xy}{\left(x^{2}+y^{2}\right)^{3/2}}$ so $\frac{∂^{2}f}{∂x∂y}=\frac{∂^{2}f}{∂y∂x}$.

3.6. $ \frac{∂^{2}f}{∂x∂y}=2xcos\left(y\right)−2ysin\left(x\right)$ and $\frac{∂^{2}f}{∂y∂x}=2xcos\left(y\right)−2ysin\left(x\right)$ so $\frac{∂^{2}f}{∂x∂y}=\frac{∂^{2}f}{∂y∂x}$.

3.7. $ \frac{∂^{2}f}{∂x∂y}=\frac{1−\left(xy\right)^{2}}{\left(1+\left(xy\right)^{2}\right)^{2}}$ and $\frac{∂^{2}f}{∂y∂x}=\frac{1−\left(xy\right)^{2}}{\left(1+\left(xy\right)^{2}\right)^{2}}$ so $\frac{∂^{2}f}{∂x∂y}=\frac{∂^{2}f}{∂y∂x}$.

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v1.0: initial version created 05/25 by Donald Campbell as part of a University of St Andrews VIP project.

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