Answers: Introduction to complex numbers

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Summary

Answers to questions relating to the guide on introduction to complex numbers.

*These are the answers to* [*Questions: Introduction to complex numbers*](../questions/qs-introtocomplexnumbers.qmd)*.*

**Please attempt the questions before reading these answers!**

## Q1

Using complex numbers, find solutions to the following equations.

1.1. Here, $x=i$ and $x=−i$ are the two solutions.

1.2. Here, $x=3i$ and $x=−3i$ are the two solutions.

1.3. Here, $x=12i$ and $x=−12i$ are the two solutions.

1.4. Here, $x=1$ and $x=−1$ are the two solutions. (Real numbers are complex numbers too!)

## Q2

For each of the complex numbers below, give their real and imaginary parts. (In this question, $a,b$ are real numbers.)

2.1. The real part of $z\_{1}$ is $Re\left(z\_{1}\right)=2$ and the imaginary part of $z\_{1}$ is $Im\left(z\_{1}\right)=3$.

2.2. The real part of $z\_{2}$ is $Re\left(z\_{2}\right)=−23$ and the imaginary part of $z\_{2}$ is $Im\left(z\_{2}\right)=32$.

2.3. The real part of $z\_{3}$ is $Re\left(z\_{3}\right)=3$ and the imaginary part of $z\_{3}$ is $Im\left(z\_{3}\right)=−3$.

2.4. The real part of $z\_{4}$ is $Re\left(z\_{4}\right)=0$ and the imaginary part of $z\_{4}$ is $Im\left(z\_{4}\right)=3$.

2.5. The real part of $z\_{5}$ is $Re\left(z\_{5}\right)=−3$ and the imaginary part of $z\_{5}$ is $Im\left(z\_{5}\right)=−2$.

2.6. The real part of $z\_{6}$ is $Re\left(z\_{6}\right)=a$ and the imaginary part of $z\_{6}$ is $Im\left(z\_{6}\right)=2b$.

2.7. The real part of $z\_{7}$ is $Re\left(z\_{7}\right)=2$ and the imaginary part of $z\_{7}$ is $Im\left(z\_{7}\right)=0$.

2.8. The real part of $z\_{8}$ is $Re\left(z\_{8}\right)=3/2$ and the imaginary part of $z\_{8}$ is $Im\left(z\_{8}\right)=2/3$.

2.9. The real part of $z\_{9}$ is $Re\left(z\_{9}\right)=22$ and the imaginary part of $z\_{9}$ is $Im\left(z\_{9}\right)=−33$.

2.10. The real part of $z\_{10}$ is $Re\left(z\_{10}\right)=333$ and the imaginary part of $z\_{10}$ is $Im\left(z\_{10}\right)=22$.

2.11. The real part of $z\_{11}$ is $Re\left(z\_{11}\right)=−2$ and the imaginary part of $z\_{11}$ is $Im\left(z\_{11}\right)=2$.

2.12. The real part of $z\_{12}$ is $Re\left(z\_{12}\right)=−2$ and the imaginary part of $z\_{11}$ is $Im\left(z\_{11}\right)=−3$.

## Q3

The complex conjugate of $z\_{1}=2+3i$ is $‾\_{1}=2−3i$.

The complex conjugate of $z\_{2}=−23+32i$ is $‾\_{2}=−23−32i$.

The complex conjugate of $z\_{3}=3−3i$ is $‾\_{3}=3+3i$.

The complex conjugate of $z\_{4}=3i$ is $‾\_{4}=−3i$.

The complex conjugate of $z\_{5}=−3−2i$ is $‾\_{5}=−3+2i$.

The complex conjugate of $z\_{6}=a+2bi$ is $‾\_{6}=a−2bi$.

The complex conjugate of $z\_{7}=2$ is $‾\_{7}=2$.

The complex conjugate of $z\_{8}=3/2+2i/3$ is $‾\_{8}=3/2−2i/3$.

The complex conjugate of $z\_{9}=22−33i$ is $‾\_{9}=22+33i$.

The complex conjugate of $z\_{10}=333+22i$ is $‾\_{10}=333+22i$.

The complex conjugate of $z\_{11}=2i−2$ is $‾\_{11}=−2i−2$.

The complex conjugate of $z\_{12}=−3i−2$ is $‾\_{12}=3i−2$.

## Q4

See [Figure 1](#fig-1) for the Argand diagram. You can notice that the complex conjugates of the complex numbers can be obtained by reflecting the point in the real axis.

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| Figure 1: An Argand diagram with the seven complex numbers $z\_{1},‾\_{1},z\_{4},‾\_{4},$ in Example 5. |

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v1.0: initial version created 10/24 by tdhc.

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