Answers: Introduction to vectors

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Summary

Answers to questions relating to the guide on introduction to vectors.

*These are the answers to* [*Questions: Introduction to vectors*](../questions/qs-introductiontovectors.qmd)*.*

**Please attempt the questions before reading these answers!**

## Q1

1.1. $\left|a\right|=\sqrt{\left(−1\right)^{2}+3^{2}}=\sqrt{1+9}=\sqrt{10}$

1.2. $\left|b\right|=\sqrt{2^{2}+4^{2}+6^{2}}=\sqrt{4+16+36}=\sqrt{56}=2\sqrt{14}$

1.3. $\left|c\right|=\sqrt{1^{2}+\left(−1\right)^{2}+4^{2}}=\sqrt{1+1+16}=\sqrt{18}=3\sqrt{2}$

1.4. $\left|d\right|=\sqrt{5^{2}+\left(−2\right)^{2}+1^{2}}=\sqrt{25+4+1}=\sqrt{30}$

1.5. $\left|e\right|=\sqrt{2^{2}+\left(−1\right)^{2}+4^{2}}=\sqrt{4+1+16}=\sqrt{21}$

1.6. $\left|f\right|=\sqrt{\left(−3\right)^{2}+6^{2}+2^{2}}=\sqrt{9+36+4}=\sqrt{49}=7$

1.7. $\left|g\right|=\sqrt{5^{2}+1^{2}+\left(\sqrt{2}\right)^{2}}=\sqrt{25+1+2}=\sqrt{28}=2\sqrt{7}$

1.8. $\left|h\right|=\sqrt{6^{2}+2^{2}+2^{2}}=\sqrt{36+4+4}=\sqrt{44}=2\sqrt{11}$

1.9. $\left|m\right|=\sqrt{\left(−3\right)^{2}+3^{2}+\left(−3\right)^{2}}=\sqrt{9+9+9}=\sqrt{27}=3\sqrt{3}$

1.10. $\left|n\right|=\sqrt{2^{2}+4^{2}+4^{2}}=\sqrt{4+16+16}=\sqrt{36}=6$

1.11. $\left|p\right|=\sqrt{8^{2}+\left(−2\right)^{2}+16^{2}}=\sqrt{64+4+256}=\sqrt{324}=18$

1.12. $\left|q\right|=\sqrt{5^{2}+\left(−2\right)^{2}+14^{2}}=\sqrt{25+4+196}=\sqrt{225}=15$

1.13. $\left|u\right|=\sqrt{7^{2}+2^{2}+\left(−1\right)^{2}}=\sqrt{49+4+1}=\sqrt{54}=3\sqrt{6}$

1.14. $\left|v\right|=\sqrt{12^{2}+9^{2}+8^{2}}=\sqrt{144+81+64}=\sqrt{289}=17$

## Q2

2.1. Find the magnitude of the vector first, so $\left|a\right|=\sqrt{\left(−2\right)^{2}+3^{2}}=\sqrt{13}$.

Then

$$\hat{a}=\frac{−2i+3j}{\sqrt{13}}=\frac{−2}{\sqrt{13}}i+\frac{3}{\sqrt{13}}j$$

2.2. Find the magnitude of the vector first, so $\left|b\right|=\sqrt{\left(−2\right)^{2}+4^{2}+\left(−6\right)^{2}}=\sqrt{56}=2\sqrt{14}$.

Then

$$\hat{b}=\frac{−2i+4j−6k}{2\sqrt{14}}=\frac{−1}{\sqrt{14}}i+\frac{2}{\sqrt{14}}j−\frac{3}{\sqrt{14}}k$$

2.3. Find the magnitude of the vector first, so $\left|c\right|=\sqrt{1^{2}+2^{2}+4^{2}}=\sqrt{21}$.

Then

$$\hat{c}=\frac{i+2j+4k}{\sqrt{21}}=\frac{1}{\sqrt{21}}i+\frac{2}{\sqrt{21}}j+\frac{4}{\sqrt{21}}k$$

2.4. Find the magnitude of the vector first, so $\left|d\right|=\sqrt{4^{2}+\left(−2\right)^{2}+3^{2}}=\sqrt{29}$.

Then

$$\hat{d}=\frac{4i−2j+3k}{\sqrt{29}}=\frac{4}{\sqrt{29}}i−\frac{2}{\sqrt{29}}j+\frac{3}{\sqrt{29}}k$$

2.5. Find the magnitude of the vector first, so $\left|e\right|=\sqrt{3^{2}+2^{2}}=\sqrt{13}$.

Then

$$\hat{e}=\frac{3i+2k}{\sqrt{13}}=\frac{3}{\sqrt{13}}i+\frac{2}{\sqrt{13}}k$$

2.6. Find the magnitude of the vector first, so $\left|f\right|=\sqrt{\left(−3\right)^{2}+1^{2}+7^{2}}=\sqrt{59}$.

Then

$$\hat{f}=\frac{−3i+j+7k}{\sqrt{59}}=−\frac{3}{\sqrt{59}}i+\frac{1}{\sqrt{59}}j+\frac{7}{\sqrt{59}}k$$

2.7. Find the magnitude of the vector first, so $\left|g\right|=\sqrt{\left(−5\right)^{2}+\left(\sqrt{2}\right)^{2}}=\sqrt{27}=3\sqrt{3}$.

Then

$$\hat{g}=\frac{−5i+\sqrt{2}k}{3\sqrt{3}}=−\frac{5}{3\sqrt{3}}i+\frac{\sqrt{2}}{3\sqrt{3}}k$$

2.8. Find the magnitude of the vector first, so $\left|h\right|=\sqrt{\left(−3\right)^{2}+1^{2}+1^{2}}=\sqrt{11}$.

Then

$$\hat{h}=\frac{−3i+j+k}{\sqrt{11}}=\frac{−3}{\sqrt{11}}i+\frac{1}{\sqrt{11}}j+\frac{1}{\sqrt{11}}k$$

2.9. Find the magnitude of the vector first, so $\left|m\right|=\sqrt{\left(−3\right)^{2}+3^{2}+\left(−3\right)^{2}}=\sqrt{27}=3\sqrt{3}$.

Then

$$\hat{m}=\frac{−3i+3j−3k}{3\sqrt{3}}=−\frac{1}{\sqrt{3}}i+\frac{1}{\sqrt{3}}j−\frac{1}{\sqrt{3}}k$$

2.10. Find the magnitude of the vector first, so $\left|n\right|=\sqrt{3^{2}+6^{2}+9^{2}}=\sqrt{126}=3\sqrt{14}$.

Then

$$\hat{n}=\frac{3i+6j+9k}{3\sqrt{14}}=\frac{1}{\sqrt{14}}i+\frac{2}{\sqrt{14}}j+\frac{3}{\sqrt{14}}k$$

2.11. Find the magnitude of the vector first, so $\left|p\right|=\sqrt{3^{2}+\left(−4\right)^{2}+\left(−5\right)^{2}}=\sqrt{50}=5\sqrt{2}$.

Then

$$\hat{p}=\frac{3i−4j−5k}{5\sqrt{2}}=\frac{3}{5\sqrt{2}}i−\frac{4}{5\sqrt{2}}j−\frac{1}{\sqrt{2}}k$$

2.12. Find the magnitude of the vector first, so $\left|q\right|=\sqrt{4^{2}+\left(−3\right)^{2}+12^{2}}=\sqrt{169}=13$.

Then

$$\hat{q}=\frac{4i−3j+12k}{13}=\frac{4}{13}i−\frac{3}{13}j+\frac{12}{13}k$$

2.13. Find the magnitude of the vector first, so $\left|u\right|=\sqrt{6^{2}+5^{2}+4^{2}}=\sqrt{77}=$.

Then

$$\hat{u}=\frac{6i+5j+4k}{\sqrt{77}}=\frac{6}{\sqrt{77}}i+\frac{5}{\sqrt{77}}j+\frac{4}{\sqrt{77}}k$$

2.14. Find the magnitude of the vector first, so $\left|v\right|=\sqrt{2^{2}+4^{2}+8^{2}}=\sqrt{84}=2\sqrt{21}$.

Then

$$\hat{v}=\frac{2i+4j+8k}{2\sqrt{21}}=\frac{1}{\sqrt{21}}i+\frac{2}{\sqrt{21}}j+\frac{4}{\sqrt{21}}k$$

## Version history and licensing

v1.0: initial version created 08/23 by Zheng Chen as part of a University of St Andrews STEP project.

* v1.1: edited 05/24 by tdhc.

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