Answers: Expected value, variance, standard deviation

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Summary

Answers to questions relating to the guide on expected value, variance, and standard deviation.

*These are the answers to* [*Questions: PMFs, PDFs, and CDFs.*](../questions/qs-expectedvariance.qmd)

**Please attempt the questions before reading these answers!**

## Q1

For each of the following valid random variables with associated probability mass function, work out the expected value and variance.

#### 1.1.

Here, $E\left(X\right)=2.8$ and $V\left(X\right)=0.76$.

#### 1.2.

Here, $E\left(X\right)=2.4$ and $V\left(X\right)=2.04$.

#### 1.3.

This is a Bernoulli trial, with probability of success $p=0.7$. From Example 2 of [Guide: Expected value, variance, standard deviation](../studyguides/expectedvariance.qmd), you can say that $E\left(X\right)=p=0.3$. The variance can be worked out to be

$$V\left(X\right)=0.3−\left(0.3\right)^{2}=0.21=\left(0.3\right)\left(0.7\right)=p\left(1−p\right).$$

#### 1.4.

Here, $E\left(X\right)=3$ and $V\left(X\right)=1$.

## Q2

For each of the following valid random variables with associated probability density function, work out the expected value andvariance.

#### 2.1.

This is the continuous uniform distribution with $a=0$ and $b=2$. From Example 2 of [Guide: Expected value, variance, standard deviation](../studyguides/expectedvariance.qmd), you can say that $E\left(X\right)=\left(a+b\right)/2=\left(0+2\right)/2=1$ and

$$V\left(X\right)=\frac{1}{12}\left(b−a\right)^{2}=\frac{1}{12}\left(2−0\right)^{2}=\frac{4}{12}=\frac{1}{3}.$$

#### 2.2.

Here, $E\left(X\right)=\frac{2}{3}$ and $V\left(X\right)=\frac{1}{6}$.

## Q3

You know that the expected value and variance of rolling one fair die are $E\left(X\right)=\frac{7}{2}$ and $V\left(X\right)=\frac{35}{12}$.

Because the roll of each die is an independent event, the random variable $Y$ of rolling seven dice is the same as

$$Y=X+X+X+X+X+X+X$$

which is seven lots of $X$.

You can use the properties of expected values and variance to get

$$\begin{matrix}E\left(Y\right)&=E\left(X+X+X+X+X+X+X\right)\\&=7⋅\frac{7}{2}=\frac{49}{2}=24.5\end{matrix}$$

and

$$\begin{matrix}V\left(Y\right)&=V\left(X+X+X+X+X+X+X\right)\\&=7⋅\frac{35}{12}=\frac{245}{12}\end{matrix}$$

## Q4

When doing this question, you need to find the two integrals

$$E\left(X\right)=\int\_{0}^{\infty }λxe^{−λx} dx  and  V\left(X\right)=\int\_{0}^{\infty }λx^{2}e^{−λx} dx.$$

You will need to use integration by parts; you should always differentiate the power of $x$ to reduce the size of the power. Use the given result to evaluate the integral when $x\rightarrow \infty $; the antiderivative should always be $0$ in this case.

## Version history and licensing

v1.0: initial version created 08/25 by tdhc.

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