Answers: Conditional probability

Sophie Chowgule

Summary

Answers to questions relating to the guide on conditional probability.

*These are the answers to* [*Questions: Conditional probability*](../questions/conditionalprobability.qmd)*.*

**Please attempt the questions before reading these answers.**

## Q1

#### 1.1.

* $P\left(A\right)=\frac{13}{52}$ (hearts)
* $P\left(B\right)=\frac{26}{52}$ (red cards)
* $P\left(A∩B\right)=\frac{13}{52}$ (red hearts)

Using the definition of conditional probability:

$$P\left(A∣B\right)=\frac{P\left(A∩B\right)}{P\left(B\right)}=\frac{13/52}{26/52}=\frac{1}{2}$$

So the probability that the card is a heart, given that it is red, is $1/2$.

#### 1.2.

You are given:

* $P\left(Piano∣Left-handed\right)=0.25$

So the probability that a randomly chosen student plays the piano, given that they are left-handed, is $0.25$.

#### 1.3.

* $P\left(A∩B\right)=0.15$
* $P\left(B\right)=0.30$

Using the definition of conditional probability:

$$P\left(A∣B\right)=\frac{P\left(A∩B\right)}{P\left(B\right)}=\frac{0.15}{0.30}=0.5$$

So the probability that an employee takes Spanish, given that they take French, is $0.5$.

#### 1.4.

* $P\left(A∩B\right)=0.25$
* $P\left(B\right)=0.40$

Using the definition of conditional probability:

$$P\left(A∣B\right)=\frac{0.25}{0.40}=\frac{5}{8}=0.625$$

So the probability that the student is sixteen, given they bring a packed lunch, is $0.625$.

## Q2

#### 2.1.

* $P\left(first green\right)=\frac{3}{5}$
* $P\left(second green∣first green\right)=\frac{2}{4}$

Using the multiplication rule:

$$P\left(both green\right)=\left(\frac{3}{5}\right)\left(\frac{2}{4}\right)=\frac{6}{20}=0.3$$

So the probability that both sweets are green is $0.3$.

#### 2.2.

* $P\left(first pass\right)=0.9$
* $P\left(second pass∣first pass\right)=0.95$

Using the multiplication rule:

$$P\left(both pass\right)=\left(0.9\right)\left(0.95\right)=0.855$$

So the probability that a box of Bayes Biscuits passes both inspections is $0.855$.

#### 2.3.

These are independent events, so

* $P\left(heads\right)=0.5$
* $P\left(roll a 6\right)=\frac{1}{6}$

Using the multiplication rule:

$$P\left(heads and 6\right)=\left(0.5\right)\left(\frac{1}{6}\right)=\frac{1}{12}$$

So the probability of getting heads and rolling a $6$ is $\frac{1}{12}$.

#### 2.4.

* $P\left(likes tea\right)=0.7$
* $P\left(likes coffee∣likes tea\right)=0.6$

Using the multiplication rule:

$$P\left(likes both\right)=\left(0.7\right)\left(0.6\right)=0.42$$

So the probability that a random person likes both tea and coffee is $0.42$.

## Q3

#### 3.1.

Given: $P\left(A\right)=0.4$, $P\left(B\right)=0.5$, $P\left(A∩B\right)=0.2$

Check:

$$P\left(A\right)P\left(B\right)=\left(0.4\right)\left(0.5\right)=0.2$$

Since $P\left(A∩B\right)=P\left(A\right)P\left(B\right)$, the events are **independent**.

#### 3.2.

Given: $P\left(A\right)=0.3$, $P\left(A∣B\right)=0.3$

Since $P\left(A∣B\right)=P\left(A\right)$, events are **independent**.

#### 3.3.

Given: $P\left(A\right)=0.5$, $P\left(B\right)=0.4$, $P\left(A∩B\right)=0.1$

Check:

$$P\left(A\right)P\left(B\right)=\left(0.5\right)\left(0.4\right)=0.2\ne 0.1=P\left(A∩B\right)$$

Since $P\left(A∩B\right)\ne P\left(A\right)P\left(B\right)$, the events are **dependent**.

#### 3.4.

Given: $P\left(A\right)=0.6$, $P\left(A∣B\right)=0.2$

Since $P\left(A∣B\right)\ne P\left(A\right)$, the events are **dependent**.

## Version history and licensing

v1.0: initial version created 05/25 by Sophie Chowgule as part of a University of St Andrews VIP project.

[This work is licensed under CC BY-NC-SA 4.0.](https://creativecommons.org/licenses/by-nc-sa/4.0/?ref=chooser-v1)