Answers: Completing the square

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Summary

Answers to questions relating to the guide on completing the square.

*These are the answers to* [*Questions: Completing the square*](../questions/qs-completingthesquare.qmd)*.*

**Please attempt the questions before reading these answers!**

## Q1

1.1. Here, $x^{2}−2x+15=\left(x−1\right)^{2}+14$, so in this question $p=−1$ and $q=14$.

1.2. Here, $y^{2}−6y+8=\left(y−3\right)^{2}−1$, so in this question $p=−3$ and $q=−1$.

1.3. Here, $x^{2}+8x+20=\left(x+4\right)^{2}+4$, so in this question $p=4$ and $q=4$.

1.4. Here, $m^{2}−26m+25=\left(m−13\right)^{2}−144$, so in this question $p=−13$ and $q=−144$.

1.5. Here, $n^{2}+6n+50=\left(m+3\right)^{2}+41$, so in this question $p=3$ and $q=41$.

1.6. Here, $x^{2}+2x+144=\left(x+1\right)^{2}+143$, so in this question $p=1$ and $q=143$.

1.7. Here, $h^{2}−3h−3=\left(h−\frac{3}{2}\right)^{2}+\frac{3}{4}$, so in this question $p=−3/2$ and $q=3/4$.

1.8. Here, $x^{2}+x−3=\left(x+\frac{1}{2}\right)^{2}−\frac{13}{4}$, so in this question $p=1/2$ and $q=−13/4$.

1.9. Here, $x^{2}−13x+43=\left(x−\frac{13}{2}\right)^{2}+\frac{3}{4}$, so in this question $p=−13/2$ and $q=3/4$.

1.10. Here, $y^{2}−8y+16=\left(y−4\right)^{2}$, so in this question $p=−4$ and $q=0$.

1.11. Here, $x^{2}+13x+9=\left(x+\frac{13}{2}\right)^{2}−\frac{133}{4}$, so in this question $p=13/2$ and $q=−133/4$.

1.12. Here, $m^{2}+3m+33=\left(m+\frac{3}{2}\right)^{2}−\frac{143}{4}$, so in this question $p=3/2$ and $q=−143/4$.

## Q2

2.1. Here, $2x^{2}−12x+14=2\left(x−3\right)^{2}−4$, so in this question $a=2$, $p=−3$ and $q=−4$.

2.2. Here, $5y^{2}−10y+4=5\left(x−1\right)^{2}−1$, so in this question $a=5$, $p=−1$ and $q=−1$.

2.3. Here, $4x^{2}+32x+68=4\left(x+4\right)^{2}+4$, so in this question $a=p=q=4$. (Or, if you prefer, $\left(2x+8\right)^{2}+4$.)

2.4. Here, $2m^{2}+2m+2=2\left(m+\frac{1}{2}\right)^{2}+\frac{3}{2}$, so in this question $a=2$, $p=1/2$ and $q=3/2$.

2.5. Here, $3x^{2}−2x+5=3\left(x−\frac{1}{3}\right)^{2}+\frac{14}{3}$, so in this question $a=3$, $p=−1/3$ and $q=14/3$.

2.6. Here, $4x^{2}−4x+1=4\left(x−\frac{1}{2}\right)^{2}$, so in this question $a=4$, $p=−1/2$ and $q=0$. (Or, if you prefer, $\left(2x−1\right)^{2}$.)

2.7. Here, $2h^{2}−3h+1=2\left(h−\frac{3}{4}\right)^{2}−\frac{1}{8}$, so in this question $a=2$, $p=−3/4$ and $q=−1/8$.

2.8. Here, $3x^{2}+5x+2=3\left(x+\frac{5}{6}\right)^{2}−\frac{3}{36}$, so in this question $a=3$, $p=5/6$ and $q=−3/36$.

## Q3

Using your working from Q1 and Q2, solve the following quadratic equations.

3.1. You worked out in 1.2 that $y^{2}−6y+8=\left(y−3\right)^{2}−1$. Rearranging $\left(y−3\right)^{2}−1=0$ for $y$ gives $y=3\pm 1$, so $y=2$ or $y=4$.

3.2. You worked out in 1.4 that $m^{2}−26m+25=\left(m−13\right)^{2}−144$. Rearranging $\left(y−3\right)^{2}−144=0$ for $y$ gives $y=13\pm 12$, so $y=1$ or $y=25$.

3.3. You worked out in 1.3 that $x^{2}+8x+20=\left(x+4\right)^{2}+4$. Using the fact that $\left(\pm 2i\right)^{2}=−4$ (see [Guide: Introduction to complex numbers]), rearranging $\left(x+4\right)^{2}+4=0$ for $y$ gives $y=−4\pm 2i$, so $y=−4−2i$ or $y=−4+2i$.

3.4. You worked out in 2.6 that $4x^{2}−4x+1=4\left(x−\frac{1}{2}\right)^{2}$. Rearranging $4\left(x−\frac{1}{2}\right)^{2}=0$ for $x$ gives $x=\frac{1}{2}$ (twice, see [Guide: Introduction to quadratic equations](../studyguides/introtoquadratics.qmd)).

3.5. You worked out in 2.3 that $4x^{2}+32x+68=4\left(x+4\right)^{2}+4$. Using the fact that $\left(\pm i\right)^{2}=−1$ (see [Guide: Introduction to complex numbers]), rearranging $4\left(x+4\right)^{2}+4=0$ for $x$ gives $x=−4\pm i$, so $x=−4−i$ or $x=−4+i$.

3.6. You worked out in 2.8 that $3x^{2}+5x+2=3\left(x+\frac{5}{6}\right)^{2}−\frac{3}{36}$. Rearranging $3\left(x+\frac{5}{6}\right)^{2}−\frac{3}{36}=0$ for $x$ gives $y=−\frac{5}{6}\pm \frac{1}{6}$, so $y=−1$ or $y=−2/3$.

## Version history and licensing

v1.0: initial version created 09/24 by tdhc.

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