Answers: Law of total probability and Bayes’ theorem

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Summary

Answers to questions relating to the guide on the law of total probability and Bayes’ theorem.

*These are the answers to* [*Questions: Law of total probability and Bayes’ theorem*](../questions/qs-bayestheorem.qmd)*.*

**Please attempt the questions before reading these answers.**

## Q1

#### 1.1.

You know:

* $P\left(Ward A\right)=0.4$
* $P\left(Recover∣Ward A\right)=0.8$
* $P\left(Ward B\right)=0.6$
* $P\left(Recover∣Ward B\right)=0.6$

Using the law of total probability:

$$P\left(Recover\right)=\left(\frac{4}{10}\right)\left(\frac{8}{10}\right)+\left(\frac{6}{10}\right)\left(\frac{6}{10}\right)=0.32+0.36=0.68$$

So the probability that a randomly chosen patient recovers is $0.68$.

#### 1.2.

You know:

* $P\left(Veg\right)=0.5$, $P\left(Finish∣Veg\right)=0.9$
* $P\left(Chicken\right)=0.3$, $P\left(Finish∣Chicken\right)=0.7$
* $P\left(Fish\right)=0.2$, $P\left(Finish∣Fish\right)=0.8$

Using the law of total probability:

$$P\left(Finish\right)=\left(0.5\right)\left(0.9\right)+\left(0.3\right)\left(0.7\right)+\left(0.2\right)\left(0.8\right)=0.45+0.21+0.16=0.82$$

So the probability that a randomly chosen student finishes their lunch is $0.82$.

#### 1.3.

You know:

* $P\left(F\_{1}\right)=0.2$, $P\left(Defective∣F\_{1}\right)=0.05$
* $P\left(F\_{2}\right)=0.3$, $P\left(Defective∣F\_{2}\right)=0.02$
* $P\left(F\_{3}\right)=0.5$, $P\left(Defective∣F\_{3}\right)=0.01$

Using the law of total probability:

$$P\left(Defective\right)=\left(0.2\right)\left(0.05\right)+\left(0.3\right)\left(0.02\right)+\left(0.5\right)\left(0.01\right)=0.01+0.006+0.005=0.021$$

So the probability that a randomly selected product is defective is $0.021$.

#### 1.4.

You know:

* $P\left(Home\right)=0.5$, $P\left(Complete∣Home\right)=0.7$
* $P\left(Library\right)=0.3$, $P\left(Complete∣Library\right)=0.9$
* $P\left(Café\right)=0.2$, $P\left(Complete∣Café\right)=0.6$

Using the law of total probability:

$$P\left(Complete\right)=\left(0.5\right)\left(0.7\right)+\left(0.3\right)\left(0.9\right)+\left(0.2\right)\left(0.6\right)=0.35+0.27+0.12=0.74$$

So the probability that the student completes their homework is $0.74$.

## Q2

#### 2.1.

You know:

* $P\left(D\right)=0.02$
* $P\left(Pos∣D\right)=0.95$
* $P\left(Pos∣¬D\right)=0.1$ (where $¬D$ means the person does not have the disease)
* $P\left(¬D\right)=0.98$

Using the law of total probability:

$$P\left(Pos\right)=\left(0.02\right)\left(0.95\right)+\left(0.98\right)\left(0.1\right)=0.019+0.098=0.117$$

Now applying Bayes’ theorem:

$$P\left(D∣Pos\right)=\frac{\left(0.95\right)\left(0.02\right)}{0.117}≈0.162$$

So the probability that the person has the disease, given that they test positive, is approximately $0.162$. Not a very good test!

#### 2.2.

You know:

* $P\left(Rain\right)=0.4$
* $P\left(Dry\right)=0.6$
* $P\left(F∣Rain\right)=0.8$
* $P\left(F∣Dry\right)=0.1$

Using the law of total probability:

$$P\left(F\right)=\left(0.4\right)\left(0.8\right)+\left(0.6\right)\left(0.1\right)=0.32+0.06=0.38$$

Then applying Bayes’ theorem gives:

$$P\left(Rain∣F\right)=\frac{\left(0.8\right)\left(0.4\right)}{0.38}≈0.842$$

So the probability that it actually rains in St Andrews, given that the forecast predicts rain, is approximately $0.842$.

#### 2.3.

You know:

* $P\left(A\right)=0.7$
* $P\left(B\right)=0.3$
* $P\left(F∣A\right)=0.02$
* $P\left(F∣B\right)=0.05$

Using the law of total probability:

$$P\left(F\right)=\left(0.7\right)\left(0.02\right)+\left(0.3\right)\left(0.05\right)=0.014+0.015=0.029$$

Then applying Bayes’ theorem gives:

$$P\left(B∣F\right)=\frac{\left(0.05\right)\left(0.3\right)}{0.029}≈0.517$$

So the probability that the broken biscuit came from Machine B, given that it is broken, is approximately $0.517$.

#### 2.4.

You know:

* $P\left(Red\right)=0.4$
* $P\left(Blue\right)=0.6$
* $P\left(W∣Red\right)=0.3$
* $P\left(W∣Blue\right)=0.7$

Using the law of total probability:

$$P\left(W\right)=\left(0.4\right)\left(0.3\right)+\left(0.6\right)\left(0.7\right)=0.12+0.42=0.54$$

Then applying Bayes’ theorem gives:

$$P\left(Red∣W\right)=\frac{\left(0.3\right)\left(0.4\right)}{0.54}≈0.222$$

So the probability that the sweet is red, given that it has a wrapper, is approximately $0.222$.

## Version history and licensing

v1.0: initial version created 05/25 by Sophie Chowgule as part of a University of St Andrews VIP project.

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