Answers: Vector addition and scalar multiplication

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Summary

Answers to questions relating to the guide on vector addition and scalar multiplication.

*These are the answers to* [*Questions: Addition and scalar multiplication*](../questions/qs-addandsm.qmd)*.*

**Please attempt the questions before reading these answers!**

## Q1

1.1. For the $i$ component, $4+8=12$.For the $j$ component, $5+2=7$.For the $k$ component, $7+4=11$. So the answer is $a+b=12i+7j+11k$.

1.2. $a+b=2i+3j+9k$.

1.3. $a−b=2i−11j+14k$.

1.4. You can solve this by doing addition componentwise. $i$ component: $4−\left(3+11\right)=−10$, $j$ component: $12−\left(−3−4\right)=19$, $k$ component: $−7−\left(−2+9\right)=−14$. So the answer is $−10i+19j−14k$.

## Q2

2.1. $a+b=\left[\begin{matrix}4x\\7y\\0\end{matrix}\right]$

2.2. $a−b=\left[\begin{matrix}7\\3y−2x\\−z\end{matrix}\right]$

2.3. $a+b−c=0$ or $\left[\begin{matrix}0\\0\\0\end{matrix}\right]$.

2.4. $a$.

## Q3

3.1. $3u=\left(3\right)5j+\left(3\right)6k=15j+18k$.

3.2. $−6v=\left[\begin{matrix}0\\18\\−42\end{matrix}\right]$.

3.3. $4v−3u=\left[\begin{matrix}0\\−27\\10\end{matrix}\right]$

3.4. $−2w−\left(4u−2v\right)=\left[\begin{matrix}−4\\−32\\−2\end{matrix}\right]$

## Q4

4.1. By the laws of vector addition, $\vec{AB}=\vec{AO}+\vec{OB}=−\vec{OA}+\vec{OB}$, where $\vec{OA}$ and $\vec{OB}$ are the respective coordinates of $A$ and $B$ written in vector form. You can find $\vec{AB}$ by solving the above equation. $\vec{AB}=\left[\begin{matrix}−2−3\\5−4\\7−5\end{matrix}\right]=\left[\begin{matrix}−5\\1\\2\end{matrix}\right]$

4.2.$\vec{AB}=\left[\begin{matrix}4\\6\\0\end{matrix}\right]$, $\vec{AC}=\left[\begin{matrix}−2\\−4\\−5\end{matrix}\right]$. $\vec{AB}−\vec{AC}=\left[\begin{matrix}6\\10\\5\end{matrix}\right]$. You can also calculate this by noticing $\vec{AB}−\vec{AC}=\vec{CA}+\vec{AB}=\vec{CB}$. Then $\vec{CB}=\left[\begin{matrix}6−0\\11−1\\7−2\end{matrix}\right]=\left[\begin{matrix}6\\10\\5\end{matrix}\right]$ as required.

4.3. $\vec{AB}=\vec{OB}−\vec{OA}$. $\left[\begin{matrix}1\\5\\9\end{matrix}\right]−\left[\begin{matrix}a\_{1}\\a\_{2}\\a\_{3}\end{matrix}\right]=\left[\begin{matrix}6\\7\\−2\end{matrix}\right]$. Solving this gives $A=\left(−5,−2,11\right)$.

4.4. Let $λ$ and $μ$ be scalars. $λa+μb=13i−9j$. This gives you the simultaneous equations Solving this gives $μ=3$, $λ=2$, which gives the answer $2a+3b$.

4.5. $2\left[\begin{matrix}2\\5\\z\end{matrix}\right]+3\left[\begin{matrix}−1\\−3\\4\end{matrix}\right]=\left[\begin{matrix}x\\y\\0\end{matrix}\right]$. Solving this gives $x=3$, $y=1$ and $z=−6$.

4.6. As they are parallel $a=λb$ for some real scalar $λ$. This gives the simultaneous equations Eliminating $λ$ and solving gives $x=3$.

## Version history and licensing

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* v1.1: edited 05/24 by tdhc.

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