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Latest content: Guide: Introduction to complex numbers is now available! (25/10/24)

Completing the square

ALGEBRA | SOLVING EQUATIONS | KEY SKILLS

Topics are categorised (e.g., Algebra, Solving Equations) to make it easy for students to find relevant guides.

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SUMMARY
Completing the square is an important technique to learn for dealing with quadratic expressions in a variety of places. It can help you solve quadratic equations, accurately sketch graphs of conic sections, and is widely used in integral calculus.

Before reading this guide, it is recommended that you read Guide: Introduction to quadratic equations. Optionally, you may also find it useful to read [Guide: Expanding brackets] for the purposes of checking your answers, Guide: Laws of indices for algebraic manipulation, Guide: Introduction to rearranging equations for rearranging completed squares to solve quadratics and Guide: Introduction to complex numbers for dealing with square roots of negative numbers.

On this page
- What is completing the square?
- Expressions of the form $x^2 + bx + c$
- Expressions of the form $ax^2 + bx + c$
- Solving quadratic equations
- Quick check problems
- Further reading

Other Formats
- PDF
- MS Word

Provides PDF and Word formats for offline access, supporting diverse learning needs.

Example 10

Suppose that you want to solve the quadratic equation $y^2 - 10y + 41 = 0$. You completed the square of $y^2 - 10y + 41$ in Example 3, getting $(y-5)^2 + 16$. So $(y-5)^2 + 16 = 0$; taking the 16 over gives $(y-5)^2 = -16$. Using imaginary numbers (see Guide: Introduction to complex numbers) gives

$$y - 5 = \pm 4i$$

Therefore $y = 5 + 4i$ and $y = 5 - 4i$ are the two complex solutions of $y^2 - 10y + 41 = 0$.

Each guide includes step-by-step explanations and examples to help students grasp concepts in maths and stats.

Tip

You can complete the square on the general quadratic equation $ax^2 + bx + c = 0$ to get the **quadratic formula**, which gives solutions to **any** quadratic equation: see Guide: Using the quadratic formula for more, and Proof: The quadratic formula for how it is done.

Quick check problems

You are given four quadratic expressions below. Complete the square on each of them, by giving the constants. If your answer contains rational numbers, they should be written as fractions in their simplest form.

1. $x^2 - 6x = (x+p)^2 + q$, where $p =$ ☐ and $q =$ ☐.
2. $x^2 + 8x + 1 = (x+p)^2 + q$, where $p =$ ☐ and $q =$ ☐.
3. $x^2 - 5x + 8 = (x+p)^2 + q$, where $p =$ ☐ and $q =$ ☐.
4. $2x^2 + 8x + 15 = a(x+p)^2 + q$, where $a =$ ☐, $p =$ ☐ and $q =$ ☐.

Show Answers

Practical tips and example problems are provided throughout to reinforce learning and ensure students understand each step.

Further reading

For more questions on the subject, please go to Questions: Completing the square.

Directs students to related guides and questions for deeper exploration of topics, promoting independent learning.