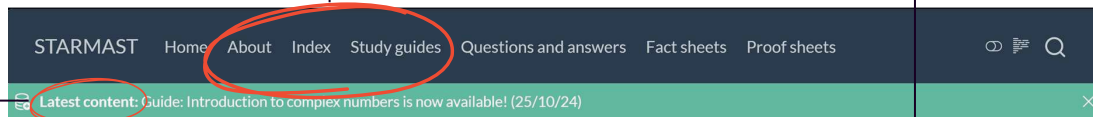


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A sidebar that allows users to jump directly to specific sections, making it easy to find exactly what they need within a guide.



Completing the square

ALGEBRA SOLVING EQUATIONS KEY SKILLS

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SUMMARY

Completing the square is an important technique to learn for dealing with quadratic expressions in a variety of places. It can help you solve quadratic equations, accurately sketch graphs of conic sections, and is widely used in integral calculus.

Before reading this guide, it is recommended that you read [Guide: Introduction to quadratic equations](#). Optionally, you may also find it useful to read [[Guide: Expanding brackets](#)] for the purposes of checking your answers, [Guide: Laws of indices](#) for algebraic manipulation, [Guide: Introduction to rearranging equations](#) for rearranging completed squares to solve quadratics and [Guide: Introduction to complex numbers](#) for dealing with square roots of negative numbers.

Example 10

Suppose that you want to solve the quadratic equation $y^2 - 10y + 41 = 0$. You completed the square of $y^2 - 10y + 41$ in Example 3, getting $(y - 5)^2 + 16$. So $(y - 5)^2 + 16 = 0$; taking the 16 over gives $(y - 5)^2 = -16$. Using imaginary numbers (see [Guide: Introduction to complex numbers](#)) gives

$$y - 5 = \pm 4i$$

Therefore $y = 5 + 4i$ and $y = 5 - 4i$ are the two complex solutions of $y^2 - 10y + 41 = 0$.

Tip

You can complete the square on the general quadratic equation $ax^2 + bx + c = 0$ to get the [quadratic formula](#), which gives solutions to **any** quadratic equation: see [Guide: Using the quadratic formula](#) for more, and [Proof: The quadratic formula](#) for how it is done.

Quick check problems

You are given four quadratic expressions below. Complete the square on each of them, by giving the constants. If your answer contains rational numbers, they should be written as fractions in their simplest form.

1. $x^2 - 6x = (x + p)^2 + q$, where $p = \square$ and $q = \square$.

2. $x^2 + 8x + 1 = (x + p)^2 + q$, where $p = \square$ and $q = \square$.

3. $x^2 - 5x + 8 = (x + p)^2 + q$, where $p = \square$ and $q = \square$.

4. $2x^2 + 8x + 15 = a(x + p)^2 + q$, where $a = \square$, $p = \square$ and $q = \square$.

Show Answers

Further reading

For more questions on the subject, please go to [Questions: Completing the square](#).

Directs students to related guides and questions for deeper exploration of topics, promoting independent learning.

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Other Formats

PDF

MS Word

Provides PDF and Word formats for offline access, supporting diverse learning needs.

Practical tips and example problems are provided throughout to reinforce learning and ensure students understand each step.